SMK PUTERIJAYA – KL (2005)
MATHEMATICS T – PAPER 1

1. If $z = 2 - 3i$, express $\frac{z+4i}{2-zi}$ in the form $a + bi$. [4]

2. If $(x-4)$ and $(2x-1)$ are factors of the expression $2x^3 - 15x^2 + ax + b$, find the values of a and b. Factorize the expression completely. [5]

3. Solve the inequality $\frac{x^2-6x+5}{x^2+2x-3} < 0$. [4]

4. Express $\frac{5x+3}{(2x+1)(x+1)^2}$ in partial fractions. [5]

5. Solve the simultaneous equations, $2^x(3^y) = 5$ and $4^x = 7(9^y)$. Give your answer correct to three decimal places. [6]

6. A geometric series has positive first term a, common ratio r and the sum to infinity S. The sum to infinity of the even-numbered terms of the series is $-\frac{1}{2}S$.

   Find the possible values of r.

   If the third term of the series is 2, show that the sum to infinity of the odd-numbered terms of the series is $\frac{81}{4}$. [7]

7. Show that if the terms involving x with powers higher than 4 are omitted, the expansion of $\sqrt{1-x}$ in ascending powers of x is given by $1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3$. [6]

   With $x = \frac{1}{64}$, estimate $\sqrt{7}$ correct to five decimal places. [4]

8. (a) Show that $\frac{d}{dx}(\tan^5 x) = 5\sec^6 x - 10\sec^4 x + 5\sec^2 x$.

   Hence, find the value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\sec^6 x - 2\sec^4 x)\,dx$. [5]

   (b) If $y = 3e^{-2x}\ln x^3$, find $\frac{dy}{dx}$ in terms of x. [3]

9. (a) Find the value of $\int_0^{\frac{\pi}{2}} 2x\sin^2 x\,dx$. [5]

   (b) Use the trapezium rule with 7 ordinates to find the approximate value of $\int_0^3 \ln(x+3)\,dx$. Give your answer correct to three decimal places.

   With the help of a graph, explain why this answer is less than the actual answer. [7]

10. The functions f and g are defined by

$$f: x \to x^2,\ x \in \mathcal{R}, \text{ and } g: x \to \begin{cases} x+1 & , x \geq 0 \\ -x & , x < 0 \end{cases}.$$

   (a) Find $f \circ g$. [3]

   (b) Find $g \circ f$. [5]

   (c) Determine if $f \circ g$ and $g \circ f$ is continuous at $x = 0$. [4]

11. Find all the points of intersection between the curves $y = x^3 - x^2 - 6x$ and $y = 3x - x^2$. Sketch these two curves on the same axes. [6]

   Find the area enclosed between these two curves. [6]

12. (a) Given that $M = \begin{pmatrix} 2 & 3 & 1 \\ -1 & 0 & 4 \\ 1 & -1 & 1 \end{pmatrix}$. Show that $M^3 - 3M^2 + 8M - 24I = 0$. [7]

   (b) Given that $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} -5 & 1 & 7 \\ 1 & 7 & -5 \\ 7 & -5 & 1 \end{pmatrix}$.

   Find AB. Hence solve the simultaneous equations,
   $-5x + y + 7z = 8$, $x + 7y - 5z = -16$ and $7x - 5y + z = 14$. [8]

MATHEMATICS T – PAPER 2

1. The angle α is such that $0 < \alpha < \frac{\pi}{2}$ and $R\cos(x+\alpha) = 84\cos x - 13\sin x$, where $R > 0$.

   (a) Find the value of R and α. [3]

   (b) Solve the equation $84\cos x - 13\sin x = 17$, for $0 \leq x \leq 2\pi$. [3]

2. Show that $\frac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta$.

   Hence, find the value of $\tan 67.5^\circ$, in the form of surds. [5]

3. Solve the differential equation $2x^2y\frac{dy}{dx} = (1+x^2)(1+y^2)$, given that $y = 1$ when $x = 1$. Give your answer in the form $y^2 = f(x)$. [6]

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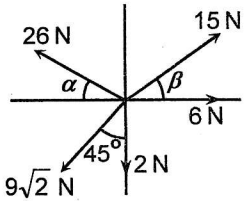
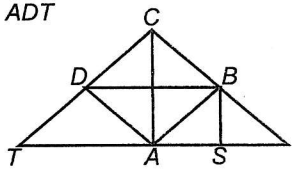
4. Five forces with magnitudes of 15 N, 26 N, $9\sqrt{2}$ N, 2 N and 6 N act on a particle as shown in the diagram.
If $\tan\alpha = \frac{5}{12}$ and $\tan\beta = \frac{3}{4}$, find the magnitude and direction of the resultant force. [6]



5. In the figure, $ABCD$ is a rhombus and CD is produced to T such that $CD = DT$. Show that the triangles BCD and ADT are congruent, and the diagonal BD is parallel to TA.
If the line parallel to AC through B is produced to meets TA produced at S, show that $\angle ASB = 90^\circ$.
State the value of $\frac{SA}{AT}$. [6]



6. In a chemical process to produce substance B from substance A, each molecule of A is changed into one molecule of B. The rate of increase in the number of molecules of b at any instant is proportional to the number of molecules of A present at that time. The number of molecules of a at the beginning of the process is N, and the number of molecules of B at the time t minutes after the start of the process is x.

By assuming x to be a continuous variable, write a differential equation to describe this chemical process.

If $x = 0$ when $t = 0$, show that $x = N(1 - e^{-kt})$, where k is a positive constant.
Sketch the graph of x against t.

If the number of molecules of A and B are equal when $t = 50$, determine the time that elapses before the number of molecules of B exceeds $\frac{99}{100}N$. [12]

7. At a particular instant, the position vector of two particles, A and B are $4\underline{i}$ m and $6\underline{i}$ m respectively. Particle A moves with a velocity of $(40\underline{i} + 30\underline{j})$ ms^{-1} and particle B moves with a velocity of $(15\underline{i} + 20\underline{j})$ ms^{-1}. Find

(a) the magnitude and direction of the velocity of A relative to B,

(b) the shortest distance between the two particle,

(c) the time taken before the two particles are at the shortest distance from each other. [9]

8. The random variable X has a Poisson distribution with $P(X = 0) = \frac{1}{2}P(X = 1)$.
Find the value of $P(X \geq 2)$.

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9. (a) In a survey conducted by the management of a factory that produces electric bulbs, it is found that 3% of the bulbs produced cannot be used. Find the minimum number of bulbs that need to be checked so that the probability of having at least one bulb that cannot be used exceeds 0.5. [6]

(b) Events A and B are such that $P(A) = \frac{2}{5}$, $P[(A \cap B') \cup (A' \cap B)] = \frac{1}{6}$ and $P(B) = \frac{1}{4}$. Find $P(A \cap B)$ and determine if events A and B are independent. [6]

10. The continuous random variable X has a cumulative distribution function defined by

$$F(x) = \begin{cases} 0 & , \quad x < -1 \\ ax - bx^2 & , -1 \leq x < 2 \\ 1 & , \quad 2 \leq x \end{cases}$$

(a) Show that $a = \frac{1}{6}$ and find the value of b.

(b) Find the value of $P(X > 1.5)$.

(c) Find the value of the median of X.

(d) Find the probability density function of X. [11]

11. A discrete random variable X has the following probability distribution,

$$P(X = x) = \begin{cases} \frac{1}{2^x} & , x = 1,2,3,4,5 \\ m & , x = 6 \\ 0 & , \text{otherwise} \end{cases}$$

where m is a constant.

(a) Determine the value of m. [2]

(b) Find the mean of X. [2]

(c) Find $P(4 \leq X \leq 6)$. [2]

(d) Find the value of $\text{Var}(4X - 2)$. [4]

12. The number of teenagers, according to age, that patronize a recreational centre for a certain period of time is indicated in the table.

Age (years)	No. of teenagers
12 –	4
13 –	10
14 –	27
15 –	110
16 –	212
17 –	238
18 –	149

(Age 12 – means age 12 and above but less than 13)

(a) Represents the data in a histogram. [3]

(b) Calculate the mean, median and semi-interquartile range for the age of the teenagers who patronize the centre. Give your answer corrects to the nearest month. [9]

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1. $w = \frac{z+4i}{2-zi} = \frac{2+i}{-1-2i} \cdot \frac{-1+2i}{-1+2i} = -\frac{4}{5}+\frac{3}{5}i$ ✱

2. Let $f(x) = 2x^3-15x^2+ax+b$

$f(4)=0$: $128-240+4a+b=0 = 4a+b-112$ — ①

$f(\frac{1}{2})=0$: $\frac{1}{4}-\frac{15}{4}+\frac{a}{2}+b=0 \Rightarrow \frac{a}{2}+b-\frac{7}{2}=0$ — ②

①−②: $\frac{7}{2}a = \frac{217}{2} \Rightarrow a=31 \quad \therefore b=-12$ ✱

$\therefore f(x) = 2x^3-15x^2+31x-12$

$= (x-4)(2x-1)(x-3)$ ✱

3. $\frac{x^2-6x+5}{x^2+2x-3} < 0 \Rightarrow \frac{(x-1)(x-5)}{(x-1)(x+3)} < 0 \quad (x \neq 1)$

$\therefore \frac{x-5}{x+3} < 0$

−	−	+	: $(x-5)$
−	+	+	: $(x+3)$
⊕	⊖	⊕	: $\frac{x-5}{x+3}$

-3 5

$\therefore \{x: -3<x<1 \text{ or } 1<x<5\}$ ✱

4. $f(x) = \frac{5x+3}{(x+1)^2(2x+1)} \equiv \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{2x+1}$

$= \frac{A(x+1)(2x+1)+B(2x+1)+C(x+1)^2}{(x+1)^2(2x+1)}$

$\therefore 5x+3 \equiv A(x+1)(2x+1)+B(2x+1)+C(x+1)^2$

$x=-1$: $-5+3 = B(-1) \Rightarrow B=2$

$x=-\frac{1}{2}$: $-\frac{5}{2}+3 = C(\frac{1}{4}) \Rightarrow C=2$

$[x^2]$: $0 = 2A+C \Rightarrow A=-1$

$\therefore f(x) = \frac{2}{2x+1} + \frac{2}{(x+1)^2} - \frac{1}{x+1}$ ✱

5. $2^x(3^y) = 5 \Rightarrow 2^x = \frac{5}{3^y}$

$4^x = 7(9^y) \Rightarrow (\frac{5}{3^y})^2 = 7(9^y)$

$\therefore 3^{4y} = \frac{25}{7} \Rightarrow y = \frac{1}{4}\left[\lg(\frac{25}{7}) / \lg 3\right]$

$\therefore y = 0.290$ (3 dp) ✱

$4^x = 7\sqrt{\frac{25}{7}} \Rightarrow x = \lg\sqrt{175} / \lg 4 = 1.863$ ✱

6. $-\frac{1}{2}S = ar+ar^3+\ldots$: $-\frac{1}{2}(\frac{a}{1-r}) = \frac{ar}{1-r^2}$

$\therefore -\frac{1}{2} = \frac{r}{1+r} \Rightarrow r = -\frac{1}{3}$ ✱

$ar^2 = 2$: $a(\frac{1}{9}) = 2 \Rightarrow a = 18$

$a+ar^2+ar^4+\ldots = 18 / 1-(-\frac{1}{3})^2 = \frac{81}{4}$ ✱

7. $(1-x)^{\frac{1}{2}} \approx 1+\frac{1}{2}(-x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(-x)^2+$

$\frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{3!}(-x)^3 = 1-\frac{1}{2}x-\frac{1}{8}x^2-\frac{1}{16}x^3$ ✱

$f(\frac{1}{64}) = (\frac{63}{64})^{\frac{1}{2}} = \frac{3}{8}\sqrt{7}$

$\therefore \sqrt{7} = \frac{8}{3}\left[1-\frac{1}{2}(\frac{1}{64})-\frac{1}{8}(\frac{1}{64})^2-\frac{1}{16}(\frac{1}{64})^3+\ldots\right]$

$= 2.64575$ (5 dp) ✱

8. (a) $\frac{d}{dx}[\tan^5 x] = 5\tan^4 x \cdot \sec^2 x$

$= 5(\sec^2 x-1)^2 \cdot \sec^2 x$

$= 5(\sec^4 x - 2\sec^2 x+1)\sec^2 x$

$= 5\sec^6 x - 10\sec^4 x + 5\sec^2 x$ ✱

$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} [\sec^6 x - 2\sec^4 x]\,dx$

$= \int_{\pi/4}^{\pi/3} \frac{1}{5}\frac{d}{dx}[\tan^5 x]\,dx - \int_{\pi/4}^{\pi/3} \sec^2 x\,dx$

$= \frac{1}{5}[\tan^5 x]_{\pi/4}^{\pi/3} - [\tan x]_{\pi/4}^{\pi/3}$

$= \frac{1}{5}(9\sqrt{3}-1) - (\sqrt{3}-1) = \frac{4}{5}(\sqrt{3}+1)$ ✱

(b) $y = 3e^{-2x}\ln x^3 = 9e^{-2x}\ln x$

$\frac{dy}{dx} = -18e^{-2x}\ln x + \frac{9e^{-2x}}{x}$ ✱

9. (a) $\int_0^{\pi/2} 2x\sin^2 x\,dx = \int_0^{\pi/2} x(1-\cos 2x)\,dx$

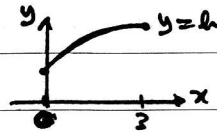
$= [x(x-\frac{1}{2}\sin 2x)]_0^{\pi/2} - \int_0^{\pi/2}(x-\frac{1}{2}\sin 2x)\,dx$

$= \frac{\pi}{2}(\frac{\pi}{2}-0) - [\frac{x^2}{2}+\frac{1}{4}\cos 2x]_0^{\pi/2}$

$= \frac{\pi^2}{4} - [\frac{\pi^2}{8}+\frac{1}{4}(-1)-\frac{1}{4}] = \frac{1}{8}(\pi^2+4)$ ✱

(b) $\int_0^3 \ln(x+3)\,dx = \frac{1}{2}(\frac{1}{2})[\ln 3+\ln 6+$

$2(\ln 3.5+\ln 4+\ln 4.5+\ln 5+\ln 5.5)] = 4.451$ ✱



Graph is concave downwards

$\Rightarrow$ graph higher than trapeziums

$\Rightarrow$ actual answer > 4.451. ✱

10. (a) $fg(x) = \begin{cases} f(x+1), & x \geq 0 \\ f(-x), & x<0 \end{cases} = \begin{cases} (x+1)^2, & x \geq 0 \\ x^2, & x<0 \end{cases}$ ✱

(b) $gf(x) = g(x^2) = x^2+1, \quad x \in \mathbb{R}$. ✱

(c) $gf(0) = 0^2+1 = 1 = \lim_{x\to 0} gf(x)$

$\therefore gf$ is continuous at $x=0$ ✱

$fg(0) = (0+1)^2 = 1$

$\lim_{x\to 0^-} fg(x) = 0^2 = 0$

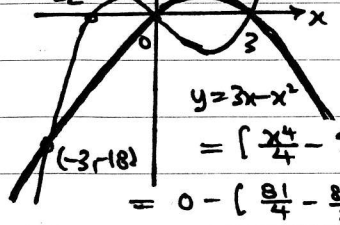
$fg(0) \neq \lim_{x\to 0^-} fg(x)$

$\therefore fg$ is not continuous at $x=0$ ✱

11. $x^3-x^2-6x = 3x-x^2 \Rightarrow x^3-9x=0$

$\therefore x(x^2-9)=0 \Rightarrow x=0$ or ± 3

$\therefore$ intersection points $= (0,0), (3,0), (-3,-18)$ ✱



Area $= \int_{-3}^{0}(x^3-9x)\,dx$

$+ \int_0^3 (9x-x^3)\,dx$

$= [\frac{x^4}{4}-\frac{9x^2}{2}]_{-3}^{0} + [\frac{9x^2}{2}-\frac{x^4}{4}]_0^3$

$= 0-[\frac{81}{4}-\frac{81}{2}]+[\frac{81}{2}-\frac{81}{4}]-0 = \frac{81}{2}$ ✱

12. (a) $M^2 = \begin{bmatrix} 2 & 3 & 1 \\ -1 & 0 & 4 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 & 1 \\ -1 & 0 & 4 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 5 & 15 \\ 2 & -7 & 3 \\ 4 & 2 & -2 \end{bmatrix}$

$\therefore M^3 = \begin{bmatrix} 2 & 3 & 1 \\ -1 & 0 & 4 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 5 & 15 \\ 2 & -7 & 3 \\ 4 & 2 & -2 \end{bmatrix} = \begin{bmatrix} 14 & -9 & 37 \\ 14 & 3 & -23 \\ 4 & 14 & 16 \end{bmatrix}$

$M^3-2M^2+8M-24I = \begin{bmatrix} 14 & -9 & 37 \\ 14 & 3 & -23 \\ 4 & 14 & 10 \end{bmatrix} - 3\begin{bmatrix} 2 & 5 & 15 \\ 2 & -7 & 3 \\ 4 & 2 & -2 \end{bmatrix}$

$+ 8\begin{bmatrix} 2 & 3 & 1 \\ -1 & 0 & 4 \\ 1 & -1 & 1 \end{bmatrix} - 24\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$ ✱

(b) $AB = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}\begin{bmatrix} -5 & 1 & 7 \\ 1 & 7 & -5 \\ -7 & -5 & 1 \end{bmatrix} = \begin{bmatrix} 18 & 0 & 0 \\ 0 & 18 & 0 \\ 0 & 0 & 18 \end{bmatrix}$ ✱

$\left.\begin{array}{l} -5x+y+7z=8 \\ x+7y-5z=-16 \\ 7x-5y+z=14 \end{array}\right\}$ $B\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ -16 \\ 14 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{18}A\begin{pmatrix} 8 \\ -16 \\ 14 \end{pmatrix}$

$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \Rightarrow x=1, y=-1, z=2$ ✱

1. (i) $84\cos x - 13\sin x = R\cos(x+\alpha) = f(x)$

$= R\cos x\cos\alpha - R\sin x\sin\alpha$

$\therefore R\cos\alpha = 84,\ R\sin\alpha = 13$ | $R\left(\frac{13}{85}\right) = 13$

$\therefore \alpha = \tan^{-1}\left(\frac{13}{84}\right) = 0.154$ | $\therefore R = 85$

(ii) $f(x) = 17$: $\cos(x + 0.154) = \frac{17}{85}$

$\therefore x + 0.154 = 4.914,\ 1.369$

$\therefore x = 4.760,\ 1.215$

2. $\tan\theta = \frac{s}{c}\cdot\frac{2c}{2c} = \frac{\sin 2\theta}{1+\cos 2\theta}$

$\therefore \tan 67\tfrac{1}{2}^\circ = \frac{\sin 135^\circ}{1+\cos 135^\circ} = \frac{1}{\sqrt{2}} \Big/ 1 - \frac{1}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}-1}\cdot\frac{\sqrt{2}+1}{\sqrt{2}+1} = \sqrt{2}+1$

3. $2x^2y\frac{dy}{dx} = (1+x^2)(1+y^2)$

$\therefore \int_1^y \frac{2y}{1+y^2}\,dy = \int_1^x \left[\frac{1}{x^2}+1\right]dx$

$\therefore \left[\ln(1+y^2)\right]_1^y = \left[-\frac{1}{x}+x\right]_1^x$

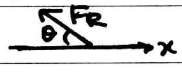
$\therefore \ln\left(\frac{1+y^2}{2}\right) = \left(x-\frac{1}{x}\right)-(-1+1)$

$\therefore \frac{1}{2}(1+y^2) = e^{x-\frac{1}{x}} \Rightarrow y^2 = 2e^{x-\frac{1}{x}}-1$

4. $F_R = 15\left(\frac{1}{5}\right)\left(\frac{4}{3}\right) + 26\left(\frac{1}{13}\right)\left(\frac{-12}{5}\right) + 9\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)\left(\frac{-1}{-1}\right)$

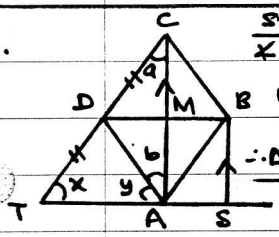
$+ 2\left(\frac{0}{-1}\right) + 6\left(\frac{1}{0}\right) = \left(\frac{-15}{8}\right)$

$= \sqrt{225+64}\,N,\ \tan\theta = \frac{8}{15}$



$= 17N,\ \tan\theta = \frac{8}{15}$

5.



Statements	Reasons
$\angle BCD = \angle ADT$	Corresponding Angles ($AD \parallel BC$)
$BC = CD = DA$	Rhombus
$CD = DT$	Given
$\therefore \Delta BCD \equiv \Delta ADT$	SAS Theorem
$a = b$	$CD = DA$
$x = y$	$DT = DA$
$\therefore a + x + y + b = 180^\circ$	Sum of angles in Δ
$\therefore b + y = 90^\circ$	
$\therefore \angle ASB = y + b = 90^\circ$	Corresponding Angles. ($AC \parallel SB$)

$\therefore SA = BM$ (Parallelogram ASBM) $BD \parallel SAT$

$BM = \frac{1}{2}BD$ (Rhombus)

$BD = AT$ (Parallelogram TABD)

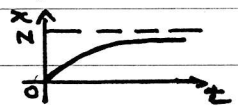
$\therefore \frac{SA}{AT} = \frac{\frac{1}{2}BD}{BD} = \frac{1}{2}$

6. $\frac{dx}{dt} \propto (N-x) \Rightarrow \frac{dx}{dt} = k(N-x)$

$\int_0^x \frac{dx}{N-x} = \int_0^t k\,dt \Rightarrow -\left[\ln(N-x)\right]_0^x = \left[kt\right]_0^t$

$\therefore \ln\left[\frac{N-x}{N}\right] = -kt \Rightarrow 1 - \frac{x}{N} = e^{-kt}$

$\therefore x = N(1-e^{-kt})$



$t = 50$: $\ln\left[\frac{N-\frac{N}{2}}{N}\right] = -50k$

$\therefore k = \frac{1}{50}\ln 2$

For $x > \frac{99}{100}N$: $t > -\frac{50}{\ln 2}\ln\left(\frac{1}{100}\right) = 100\frac{\ln 10}{\ln 2}$

7. (i) $\underline{v}_{AB} = \begin{pmatrix}40\\20\end{pmatrix} - \begin{pmatrix}15\\20\end{pmatrix} = \begin{pmatrix}25\\10\end{pmatrix} = 5\begin{pmatrix}5\\2\end{pmatrix}$

$= 5\sqrt{29}\ ms^{-1}$, to $\begin{pmatrix}5\\2\end{pmatrix}$

$= 5\sqrt{29}\ ms^{-1}$, to N 68.2° E

(ii) For shortest distance: $\left[\begin{pmatrix}0\\-2\end{pmatrix} + t\begin{pmatrix}25\\10\end{pmatrix}\right]\cdot\begin{pmatrix}25\\10\end{pmatrix} = 0$

$\therefore -20 + t(625+100) = 0 \Rightarrow t = \frac{4}{145}\ s$

$\therefore$ Shortest distance $= \left|\begin{pmatrix}0\\-2\end{pmatrix} + \frac{4}{145}\begin{pmatrix}25\\10\end{pmatrix}\right|$

$= \begin{pmatrix}0.8\\-1.68\end{pmatrix} = \sqrt{0.8^2 + (-1.68)^2} = 1.861\ m$

(iii) Time taken $= \frac{4}{145}\ s$

8. $P(X=0) = \frac{1}{2}P(X=1)$: $e^{-\lambda} = \frac{1}{2}\left[e^{-\lambda}(\lambda)\right]$

$\therefore \lambda = 2$

$\therefore P(X \geq 2) = 1 - e^{-2}[1+2] = 0.5940$

9. (a) X = no. of faulty bulbs, $X \sim B(n, 0.03)$

$P(X \geq 1) > 0.5$: $P(X=0) < 0.5$

$0.97^n < 0.5 \Rightarrow n\lg 0.97 < \lg 0.5$

$\therefore n > 22.76$ $\therefore n_{min} = 23$

(b) $P[(A\cap B')\cup(A'\cap B)] = \frac{1}{6}$

$\therefore P(A) - P(A\cap B) + P(B) - P(A\cap B) = \frac{1}{6}$

$\therefore P(A\cap B) = \frac{1}{2}\left[\frac{2}{5}+\frac{1}{4}-\frac{1}{6}\right] = \frac{29}{120}$

$P(A).P(B) = \left(\frac{2}{5}\right)\left(\frac{1}{4}\right) = \frac{1}{10}$ $[\neq P(A\cap B)]$

$\therefore$ A, B are not independent

10. (i) $F(-1) = 0$: $-a - b = 0 \Rightarrow b = -a$

$F(2) = 1$: $2a - 4b = 1 = 2a + 4a$

$\therefore a = \frac{1}{6}$ $\therefore b = -\frac{1}{6}$

(ii) $P(X > 1\frac{1}{2}) = 1 - F(\frac{3}{2}) = 1 - \frac{1}{6}\left[\frac{3}{2}+\frac{9}{4}\right] = \frac{3}{8}$

(iii) $P(X<m) = \frac{1}{2}$: $\frac{1}{2} = \frac{1}{6}(m+m^2)$

$\therefore m^2 + m - 3 = 0 \Rightarrow m = \frac{-1\pm\sqrt{1+12}}{2}$

$\therefore m = \frac{1}{2}(-1+\sqrt{13})$ $(\because -1 < m < 2)$

(iii) $f(x) = \frac{d}{dx}[F(x)] = \begin{cases}\frac{1}{6}(1+2x) & -1 \leq x < 2\\ 0 & \text{otherwise}\end{cases}$

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11. (i) $\Sigma P(X=x)=1$:

$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+m=1 \Rightarrow m=\frac{1}{32}$ #

(ii) $\bar{x} = 1(\frac{1}{2})+2(\frac{1}{4})+3(\frac{1}{8})+4(\frac{1}{16})+5(\frac{1}{32})$
$+6(\frac{1}{32}) = \frac{63}{32}$ #

(iii) $P(4 \le X \le 6) = \frac{1}{16}+\frac{1}{32}+\frac{1}{32} = \frac{1}{8}$ #

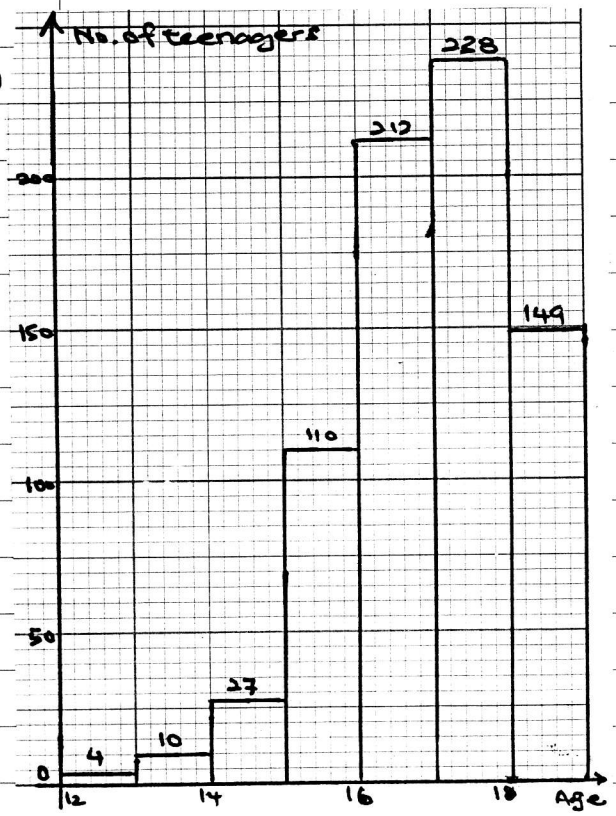
(iv) $E(X^2) = 1^2(\frac{1}{2})+2^2(\frac{1}{4})+3^2(\frac{1}{8})+4^2(\frac{1}{16})$
$+5^2(\frac{1}{32})+6^2(\frac{1}{32}) = 5\frac{17}{32}$

$\therefore Var(X) = 5\frac{17}{32}-(\frac{63}{32})^2 = \frac{1695}{1024}$

$\therefore Var(4X-2) = 16(\frac{1695}{1024}) = \frac{1695}{64}$ #

12.

(i)



(ii) $\bar{x} = \frac{\Sigma fx}{n} = \frac{12701}{750} = 16y\ 11 months$ #

$m = 17+\frac{375-363}{228}(1) = 17y\ 1 month$ #

$SIQR = \frac{1}{2}\left[(17+\frac{562.5-363}{228})-(16+\frac{187.5-151}{212})\right]$
$= 10$ months. #

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