SMK MUNSHI SULAIMAN, BATU PAHAT (2005)
MATHEMATICS T (PAPER 1)

1. Function f is defined by $f: x \to x^2 + 4x + 5, x \geq -2$.
Find the inverse of f and state its domain and range. [5]

2. The complex number is given by $a = 1 + i$. Express $a + \frac{1}{a}$ in the form of $p + qi$, where p and q are real numbers. Find the modulus and argument of $a + \frac{1}{a}$. [5]

3. Sketch in the same diagram, the graphs of $y = |5x - 7|$ and $y = 2x + 1$.
Hence, solve the inequality $|5x - 7| > 2x + 1$. [6]

4. (a) Given that $y = x^2 \ln x$. Find the value of $\frac{dy}{dx}$ when $x = c$.

(b) Given that $\tan x + \tan y = 3$. Find the value of $\frac{dy}{dx}$ when $x = \frac{\pi}{4}$. [6]

5. Express $\frac{1}{(3k-2)(3k+1)}$ in partial fractions. Hence, find an expression for $S_n = \sum_{k=1}^{n} \frac{1}{(3k-2)(3k+1)}$, and find the value of S_n when $n \to \infty$. [7]

6. If $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$, show that $\mathbf{A}^2 - 5\mathbf{A} + \mathbf{I} = 0$, where $\mathbf{I}$ is the 2 x 2 identity matrix.
Hence, find $\mathbf{A}^{-1}$ and $\mathbf{A}^3$ by using the above equation. [6]

7. (a) Show that $\frac{1}{3}$ is a zero of the polynomial $P(x) = 3x^3 - 4x^2 - 5x + 2$.
Hence, find the set of values of x such that $P(x) > 0$. [5]

(b) When a polynomial $f(x)$ is divided by $(x + 1)$ and $(x - 3)$, the remainders are 2 and 4 respectively. Find the remainder when $f(x)$ is divided by $(x^2 - 2x - 3)$. [4]

8. Given that $f(x) = \frac{1}{\sqrt{1+\sqrt{x}}}$, and integral $\int_0^1 f(x)\,dx$ is denoted by I.

(a) Using the trapezium rule with 4 trapezia of equal width, obtain an approximation I_1 to the value of I, giving 3 decimal places in your answer. [4]

(b) Using the substitution $u = 1 + \sqrt{x}$, show that the exact value of I is $\frac{4}{3}(2 - \sqrt{2})$. [5]

9. Given that $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 6 \end{pmatrix}$.

(a) Find $|\mathbf{A}|$ and $adj(\mathbf{A})$.

(b) Hence, find $\mathbf{A}^{-1}$ and solve the following simultaneous equations,
$x + 2y + 3z = 9$, $2x + 3y + 4z = 11$ and $x + 5y + 6z = 21$. [12]

10. (a) Find the equations of the lines that are inclined at 45° to the line $3x - 2y + 1 = 0$ and passing through the point $(3, -2)$. [5]

(b) A and B are points on the line $5x - 12y = 6$. The perpendicular distances of A and B from the line $3x - 4y + 2 = 0$ are 4 units respectively. B and O are on the same side of the line $3x - 4y + 2 = 0$, whereas A is on the opposite side of the line. Find the coordinates of A and B. [7]

11. The line that passes through $P(p^2, 2p)$ on the curve $y^2 = 4x$ and $Q(2, 0)$ intersects the curve again at R. Find the coordinates of R in terms of p.

Show that when p varies,

(i) the mid-point of PR lies on the curve $y^2 = 2x - 4$.

(ii) the point of intersection of the tangent at P and the tangent at R lies on a line parallel to the y–axis. [10]

12. A curve has equation $y = \frac{(x-3)^2}{(x-1)(x-9)}$.

(a) State the asymptotes for this curve. [3]

(b) Show that the curve does not exist for $0 < y < \frac{3}{4}$. [3]

(c) Find the coordinates of the turning points and determine the nature of these points. [5]

(d) Sketch the curve, showing clearly all the asymptotes and the turning points. [3]

MATHEMATICS T (PAPER 2)

1. By writing $5\cos x + 3\sin x$ in the form $R\cos(x - 1)$.
Hence, find the maximum and minimum values of $5\cos x + 3\sin x$ and find the values of x, between 0° and 360°, where the maximum and minimum values occur. [8]

2. The forces $(5\underline{i} + 4\underline{j})$ N, $(3\underline{i} + 6\underline{j})$ N and $(-4\underline{i} - 7\underline{j})$ N act at a point. Calculate the magnitude of the resultant force, and the cosine of the angle between the resultant force and the unit vector $\underline{i}$. [5]

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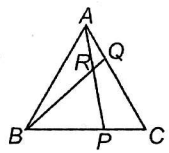
3. Derive the formula $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$.

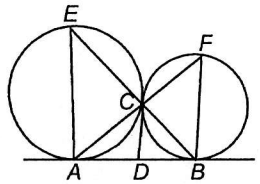
Hence, prove that $\sin x + \sin 3x + \sin 5x + \sin 7x = 16\sin x\cos^2 x\cos^2 2x$. [8]

4. (a) In the given diagram, ABC is an equilateral triangle and $BP = CQ$. Prove that
(i) $AP = BQ$.
(ii) $\angle ARB = 120°$.



(b) Diagram shows two circles touching one another externally at the point C. AB is a common tangent to the two circles. Tangent at C meets AB at D. BC produced to E and AC is produced to F, where E and F are on the circumference of the circles.

Show that
(i) $AD = BD$ (ii) $\angle AFB + \angle AEB = 90°$. [8]



5. A boy is cycling on a straight and level road, heading south. When his speed is 8 ms^{-1}, the wind seems to be blowing from the east. When his speed is 12 ms^{-1}, the wind seems to be blowing from the direction 150°. Calculate the actual speed and direction of the wind. [6]

6. (a) The variables x and y are related by the differential equation $\frac{dy}{dx} = \frac{2x+y+2}{2x+y-1}$.

Using the substitution $z = 2x + y$, show that $\frac{dz}{dx} = \frac{3z}{z-1}$.

Hence, solve the given differential equation. [5]

(b) The rate at which a radioactive substance changes from one form to another is k times the amount left unchanged, where k is a constant. At time $t = 0$, the amount of unchanged substance is A and the amount which has been changed at time t is v. Write down a differential equation for v, and solve it to give v in terms of A, k and t. Sketch the graph of v against t. [7]

7. The random variable X takes the values –2, 0, 2 with probabilities $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$ respectively. The random variable Y is defined by $Y = X_1 + X_2$, where X_1 and X_2 are two independent observations of X. Find the probability distribution of Y. Find Var (Y) and E(Y + 3). [7]

8. The number of accidents per week on a particular stretch of road follows a Poisson distribution with mean 1.2. Find

(a) the most likely number of traffic accidents in a week selected at random.

(b) the probability that in a 6 weeks period there are exactly 4 accidents. [6]

9. An old car is not parked inside the garage. On the morning after a rainy night, the probability that the car engine will not start is 0.4.

(i) For 5 consecutive nights, the rain falls. Find the probability that the engine will not start on at least 2 of the 5 mornings.

(ii) During a rainy season, there are 50 rainy nights. By using a suitable approximation, find the probability that the car engine will not start on less that 25 of the 50 days.

10. The number of cheques to be cashed received by a branch of a bank in 100 days are shown in the table.

No. of cheques	No. of days
0 – 199	10
200 – 399	13
400 – 599	17
600 – 799	42
800 – 999	18

(a) Calculate the mean and standard deviation for the number of cheques cashed every day.

(b) Calculate the median and the mode for the number of cheques cashed by the bank.

11. A machine is used to produce butter whose masses are normally distributed with a mean of μ g and a standard deviation of σ g. It is found that 5% of the output from this machine have a mass greater than 85 g and 25% have a mass less than 25 g. Find the values of μ and σ.

Find also the range of the masses of butter such that 75% of the masses are symmetrical about the mean.

12. (a) Prove that $E[(X-\mu)^2] = E(X^2) - [E(X)]^2$ for a continuous random variable X.

(b) The continuous random variable X has a probability density function f given by

$$f(x) = \begin{cases} k(x+3) & , -3 \le x \le 3 \\ 0 & , \text{otherwise} \end{cases}$$, where k is a constant.

(i) Show that $k = \frac{1}{8}$.

(ii) Find E(X) and Var (X).

(iii) Find the lower quartile of X.

(iv) Let $Y = aX + b$, where a and b are constants with $a > 0$.
Find the values of a and b for which E(Y) = 0 and Var (Y) = 1.

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1. $f: x \to x^2+4x+5 = (x+2)^2+1,\ x \geq -2$

Let $f^{-1}(x) = u$: $x = f(u) = (u+2)^2+1,\ u \geq -2$

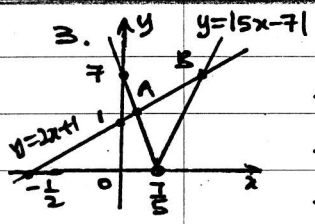
$\therefore\ u = -2+\sqrt{x-1}$

$\therefore\ f^{-1}: x \to -2+\sqrt{x-1},\ x \geq 1$

$D_{f^{-1}}: \{x : x \geq 1\}$, $R_{f^{-1}}: \{x : x \geq -2\}$

2. $z = a+\frac{1}{a} = 1+i+\frac{1}{1+i}\cdot\frac{1-i}{1-i} = \frac{3}{2}+\frac{1}{2}i$

$|z| = \frac{1}{2}\sqrt{10}$, $\arg(z) = \tan^{-1}\left(\frac{1}{3}\right) = 0.3218$

3. 

For A, B: $(5x-7)^2 = (2x+1)^2$

$\therefore\ 21x^2-74x+48 = 0$

$\therefore\ (3x-8)(7x-6) = 0$

$\therefore\ x = \frac{8}{3}$ or $\frac{6}{7}$

$\therefore$ for $|5x-7| > 2x+1$: $\{x : x < \frac{6}{7}$ or $\frac{8}{3} < x\}$

4.(a) $y = x^2\ln x \Rightarrow \frac{dy}{dx} = 2x\ln x + x^2\left(\frac{1}{x}\right)$

$x = e$: $\frac{dy}{dx} = 2e(1)+e = 3e$

(b) $\tan x + \tan y = 3 \Rightarrow \sec^2 x + \sec^2 y\frac{dy}{dx} = 0$

$x = \frac{\pi}{4}$: $1+\tan y = 3 \Rightarrow \tan y = 2$

$\therefore\ (1+1^2)+(1+2^2)\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{2}{5}$

5. $f(k) = \frac{1}{(3k-2)(3k+1)} = \frac{A}{3k-2}+\frac{B}{3k+1}$

$= \frac{A(3k+1)+B(3k-2)}{(3k-2)(3k+1)}$

$\therefore\ 1 \equiv A(3k+1)+B(3k-2)$

$3k = 2$: $1 = A(3) \Rightarrow A = \frac{1}{3}$

$[k]$: $0 = A(3)+3B \Rightarrow B = -\frac{1}{3}$

$\therefore\ f(k) = \frac{1}{3}\left[\frac{1}{3k-2}-\frac{1}{3k+1}\right]$

$S_n = \sum_{k=1}^{n} f(k) = \frac{1}{3}\left[\left(1+\frac{1}{4}+\ldots+\frac{1}{3n-2}\right)\right.$

$\left.-\left(\frac{1}{4}+\frac{1}{7}+\ldots+\frac{1}{3n-2}+\frac{1}{3n+1}\right)\right] = \frac{1}{3}\left[1-\frac{1}{3n+1}\right]$

$= \frac{n}{3n+1}$

$\lim_{n\to\infty}(S_n) = \frac{1}{3}[1-0] = \frac{1}{3}$

6. $A^2-5A+I = \begin{pmatrix}2&-1\\-5&3\end{pmatrix}\begin{pmatrix}2&-1\\-5&3\end{pmatrix}-5A+I$

$= \begin{pmatrix}9&-5\\-25&14\end{pmatrix}-\begin{pmatrix}10&-5\\-25&15\end{pmatrix}+\begin{pmatrix}1&0\\0&1\end{pmatrix} = 0$

$A^2-5A+I = 0$ —$(\times A^{-1}) \Rightarrow AI-5I+A^{-1} = 0$

$\therefore\ A^{-1} = 5\begin{pmatrix}1&0\\0&1\end{pmatrix}-\begin{pmatrix}2&-1\\-5&3\end{pmatrix} = \begin{pmatrix}3&1\\5&2\end{pmatrix}$

$A^2-5A+I = 0$ —$(\times A) \Rightarrow A^3-5A^2+A = 0$

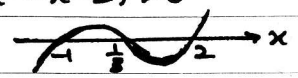
$\therefore\ A^3 = 5\begin{pmatrix}9&-5\\-25&14\end{pmatrix}-\begin{pmatrix}2&-1\\-5&3\end{pmatrix} = \begin{pmatrix}43&-24\\-120&67\end{pmatrix}$

7.(a) $P\left(\frac{1}{3}\right) = 3\left(\frac{1}{27}\right)-4\left(\frac{1}{9}\right)-5\left(\frac{1}{3}\right)+2 = 0$

$\therefore\ \frac{1}{3}$ is a zero for $P(x)$

$\therefore\ P(x) > 0 \Rightarrow (3x-1)(x^2-x-2) > 0$

$\therefore\ (3x-1)(x+1)(x-2) > 0$



$\therefore\ \{x : -1 < x < \frac{1}{3}$ or $2 < x\}$

(b) Let $f(x) = (x^2-2x-3).Q(x)+(Ax+B)$

$f(-1) = 2$: $2 = -A+B$ —①

$f(3) = 4$: $4 = 3A+B$ —②

②−①: $4A = 2$

$\therefore\ A = \frac{1}{2},\ B = \frac{5}{2}$

$\therefore$ Remainder $= \frac{1}{2}(x+5)$

8.(a) $I = \frac{1}{2}\left(\frac{1}{4}\right)\left[1+\frac{1}{\sqrt{2}}+2\left(\frac{1}{\sqrt{1+0.5}}+\frac{1}{\sqrt{1+\sqrt{0.5}}}+\frac{1}{\sqrt{1+\sqrt{0.75}}}\right)\right]$

$= 0.792$ (3dp)

(b) $\sqrt{x} = u-1$: $x = (u-1)^2 \Rightarrow \frac{dx}{du} = 2(u-1)$

$I = \int_1^2 \frac{1}{\sqrt{u}}.2(u-1)du = \left[\frac{4}{3}u\sqrt{u}-4\sqrt{u}\right]_1^2$

$= \left(\frac{8}{3}\sqrt{2}-4\sqrt{2}\right)-\left(\frac{4}{3}-4\right) = \frac{4}{3}[2-\sqrt{2}]$

9. $C(A) = \begin{bmatrix}-2&-8&7\\3&3&-3\\-1&2&-1\end{bmatrix}$

(a) $|A| = -2(1)+3(2)+(-1)(1) = 3$

$\text{adj}(A) = (C(A))^T = \begin{bmatrix}-2&3&-1\\-8&3&2\\7&-3&-1\end{bmatrix}$

(b) $A^{-1} = \frac{1}{3}\begin{bmatrix}-2&3&-1\\-8&3&2\\7&-3&-1\end{bmatrix}$

$\left.\begin{array}{l}x+2y+3z = 9\\2x+3y+4z = 11\\x+5y+6z = 21\end{array}\right\}$ $A\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}9\\11\\21\end{pmatrix}$

$\therefore\ \begin{pmatrix}x\\y\\z\end{pmatrix} = \frac{1}{3}\begin{bmatrix}-2&3&-1\\-8&3&2\\7&-3&-1\end{bmatrix}\begin{pmatrix}9\\11\\21\end{pmatrix} = \begin{pmatrix}-2\\1\\3\end{pmatrix}$ $\therefore\ x = -2,\ y = 1,\ z = 3$

10.(a) $\left|\frac{m-\frac{3}{2}}{1+m\left(\frac{3}{2}\right)}\right| = 1 \Rightarrow \frac{2m-3}{2+3m} = \pm 1$

$\therefore\ m = -5$ or $\frac{1}{5}$

$\therefore$ Eq: $\frac{y+2}{x-3} = -5$ or $\frac{1}{5} \Rightarrow y+5x = 13,\ 5y = x-13$

(b) Let $A\left(a, \frac{5a-6}{12}\right)$: $d(A) = 4$

$\left|3a-4\left(\frac{5a-6}{12}\right)+2\right|/5 = 4 \Rightarrow \frac{4}{3}a+4 = \pm 20$

$\therefore\ a = 12$ or -18 $\therefore\ A\left(12, \frac{9}{2}\right)$ or $(-18, -8)$

$D(0) = 0-0+2 = 2\ (>0)$

$a = 12$: $D = 3(12)-4\left(\frac{9}{2}\right)+2\ (>0)$

$\therefore\ A(-18, -8)$ $B\left(12, \frac{9}{2}\right)$

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11. $R(r^2, 2r)$: $m_{PR} = m_{PQ}$

$\frac{2p-2r}{p^2-r^2} = \frac{2p}{p^2-2} \Rightarrow \frac{2}{p+r} = \frac{2p}{p^2-2}$

$\therefore\ pr = -2 \Rightarrow R\left(\frac{4}{p^2}, -\frac{4}{p}\right)$ ✱

(i) Let $M(x,y)$: $x = \frac{p^2+r^2}{2}$, $y = p+r$

$\therefore\ 2x = (p+r)^2 - 2pr = y^2 - 2(-2)$

$\therefore\ y^2 = 2x - 4$ (locus of M) ✱

(ii) $y^2 = 4x$: $2y\frac{dy}{dx} = 4 \Rightarrow \frac{dy}{dx} = \frac{2}{y}$

P(tangent): $\frac{y-2p}{x-p^2} = \frac{2}{2p} \Rightarrow py = x + p^2$ — ①

R(tangent): $ry = x + r^2$ — ②

① − ② : $(p-r)y = p^2 - r^2 \Rightarrow y = p+r$

$\therefore\ x = r(p+r) - r^2 = pr = -2$

$\therefore$ Locus ($x=-2$) is a line // to y-axis ✱

12. $y = \frac{(x-3)^2}{(x-1)(x-9)} = 1 + \frac{1}{2}\left[\frac{9}{x-9} - \frac{1}{x-1}\right]$

(a) asymptotes : $y=1$, $x=9$, $x=1$ ✱

(b) $y(x^2-10x+9) = x^2-6x+9$

$\therefore\ (y-1)x^2 + (6-10y)x + (9y-9) = 0$

For $x \in \mathbb{R}$: $2^2(3-5y)^2 - 4(y-1)(9)(y-1) \geq 0$

$\therefore\ (3-5y+3y-3)(3-5y-3y+3) \geq 0$

$\therefore\ -4y(3-4y) \geq 0 \Rightarrow y \leq 0$ or $\frac{3}{4} \leq y$

$\therefore\ y \notin (0, \frac{3}{4})$ ✱

(c) $\frac{dy}{dx} = \frac{1}{2}\left[\frac{-9}{(x-9)^2} + \frac{1}{(x-1)^2}\right] = \frac{-4(x+3)(x-3)}{(x-9)^2(x-1)^2}$

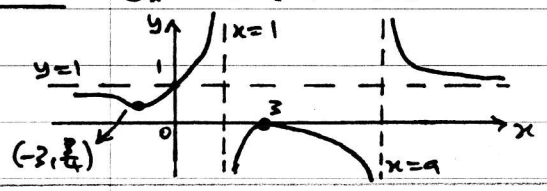
$\frac{d^2y}{dx^2} = \frac{1}{2}\left[\frac{18}{(x-9)^3} - \frac{2}{(x-1)^3}\right]$

For $\frac{dy}{dx} = 0$: $x = \pm 3$

$\therefore$ turning points $= (3,0), (-3, \frac{3}{4})$ ✱

$x=3$: $\frac{d^2y}{dx^2} = \frac{1}{2}\left[\frac{18}{216} - \frac{2}{8}\right] (<0)$ $\therefore (3,0)$ max ✱

$x=-3$: $\frac{d^2y}{dx^2} = \frac{1}{2}\left[\frac{18}{(-12)^3} + \frac{2}{64}\right] (>0)$ $\therefore (-3, \frac{3}{4})$ min ✱



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1. $f(x) = 5\cos x + 3\sin x = R\cos(x-\alpha)$

$= R\cos x\cos\alpha + R\sin x\sin\alpha$

$\therefore\ R\cos\alpha = 5$, $R\sin\alpha = 3$ | $\therefore\ R\left(\frac{3}{\sqrt{34}}\right) = 3$

$\therefore\ \tan\alpha = \frac{3}{5} \Rightarrow \alpha = 31.0°$ | $\therefore\ R = \sqrt{34}$

$\therefore\ f(x) = \sqrt{34}\cos(x - 31.0°)$ ✱

$f(x)_{max} = \sqrt{34}$ ✱, $f(x)_{min.} = -\sqrt{34}$ ✱

$f(x) = \sqrt{34}$: $\cos(x-31.0°) = 1 \Rightarrow x = 31.0°$ ✱

$f(x) = -\sqrt{34}$: $\cos(x-31.0°) = -1 \Rightarrow x = 211.0°$ ✱

2. $F_R = \begin{pmatrix}5\\4\end{pmatrix} + \begin{pmatrix}3\\6\end{pmatrix} + \begin{pmatrix}-4\\-7\end{pmatrix} = \begin{pmatrix}4\\3\end{pmatrix}$

$\therefore\ |F_R| = \sqrt{3^2+4^2} = 5N$ ✱

$\cos\theta = \frac{1}{5}\begin{pmatrix}4\\3\end{pmatrix} \cdot \begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{5}(4) = \frac{4}{5}$ ✱

3. $\sin(P+Q) = \sin P\cos Q + \cos P\sin Q$

$\sin(P-Q) = \sin P\cos Q - \cos P\sin Q$

$\therefore\ \sin(P+Q) + \sin(P-Q) = 2\sin P\cos Q$

Let $P+Q = A$, $P-Q = B$:

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ ✱

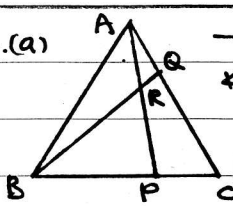
$\sin x + \sin 3x + \sin 5x + \sin 7x$

$= 2\sin 2x\cos x + 2\sin 6x\cos x$

$= 2\cos x\,[2\sin 4x\cos 2x]$ $[\sin 4x = 2\cos 2x\sin 2x]$

$= 8\cos x\sin 2x\cos^2 2x = 16\sin x\cos^2 x\cos^2 2x$ ✱

4.(a)



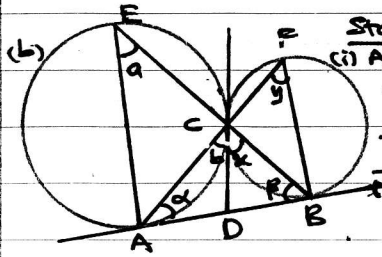
Statements	Reasons
$AB = BC$, $\angle ABP = \angle BCQ = 60°$	ΔABC is equilateral
$BP = CQ$	Given
$\Delta ABP \equiv \Delta BCQ$	By SAS

$\therefore\ AP = BQ$ ✱ $\angle BAP = \angle CBQ = \theta$

(ii) $\therefore\ \angle ABR = 60° - \angle CBQ = 60° - \theta$

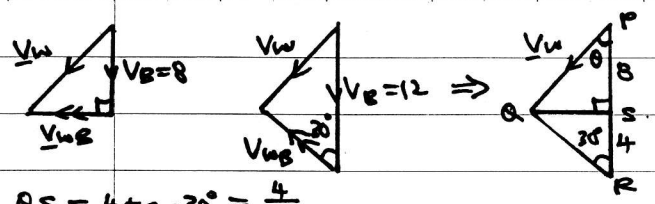
$\therefore\ \angle ABR = 180° - \angle BAP - \angle ABR = 120°$ ✱

(b)



Statements	Reasons
(i) $AD = CD$	equal tangent
$CD = BD$	equal tangent
$\therefore\ AD = BD$ ✱	
(ii) $a = b$	Angles of alternate segment.
$x = y$	Angles of alternate segment.
$b = \alpha$, $x = \beta$	Isosceles Δ
$b + \alpha + x + \beta = 180°$	Sum of angles in Δ
$\therefore\ 2b + 2x = 180°$	
$\therefore\ a + y = 90°$	
$\therefore\ \angle AEB + \angle AFB = 90°$ ✱	

5. $\underline{V}_{WB} = \underline{V}_W - \underline{V}_B \Rightarrow \underline{V}_W = \underline{V}_B + \underline{V}_{WB}$



$QS = 4\tan 30° = \frac{4}{\sqrt{3}}$

$|\underline{V}_W| = \left[8^2 + \frac{16}{3}\right]^{1/2} = \frac{4}{3}\sqrt{39}\ ms^{-1}$ ✱

$\theta = \tan^{-1}\left(\frac{4}{\sqrt{3}}/8\right) = 16.1°$

Direction : from N 16.1° E ✱

6.(a) $z = 2x+y$: $\frac{dy}{dx} = \frac{2x+y+2}{2x+y-1}$

$\therefore \frac{dz}{dx} - 2 = \frac{z+2}{z-1} \Rightarrow \frac{dz}{dx} = \frac{3z}{z-1}$ ✱

$\therefore \int (1 - \frac{1}{z})dz = \int 3\,dx \Rightarrow z - \ln z = 3x + c$

$2x+y - \ln(2x+y) = 3x + c$

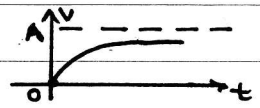
$\therefore 2x+y = e^{y-x-c} = Ae^{y-x}$ ✱ $[A = e^{-c}]$

(b) $\frac{dV}{dt} = k(A-V) \Rightarrow \int_0^V \frac{dV}{A-V} = \int_0^t k\,dt$

$\therefore \left[-\ln(A-V)\right]_0^V = k[t]_0^t$

$\therefore \ln\left(\frac{A-V}{A}\right) = -kt$

$\therefore V = A(1-e^{-kt})$ ✱



7. $P(Y=-4) = (\frac{1}{4})^2 = \frac{1}{16} = P(Y=4)$

$P(Y=-2) = (\frac{1}{4})(\frac{1}{2})(2) = \frac{1}{4} = P(Y=2)$

$P(Y=0) = (\frac{1}{4})^2(2) + (\frac{1}{2})^2 = \frac{3}{8}$

y	-4	-2	0	2	4
P(Y=y)	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

✱

$E(Y) = \frac{1}{16}(-4-8+0+8+4) = 0$

$E(Y^2) = \frac{1}{16}[16+16+0+16+16] = 4$

$\therefore Var(Y) = 4 - 0^2 = 4$ ✱

$E(Y+3) = E(Y) + 3 = 0 + 3 = 3$ ✱

8. X = no. of acc. per week, $X \sim P_o(1.2)$

(a) $P_{k+1} > P_k$ for $k < 1.2 - 1 \Rightarrow k = 0$

$\therefore P_0 < P_1 > P_2 > P_3 \ldots$

most likely no. of acc = 1 ✱

(b) Y = no. of acc. in 6 weeks, $Y \sim P_o(7.2)$

$P(Y=4) = e^{-7.2}\left(\frac{7.2^4}{4!}\right) = 0.0836$ ✱

9. Let X = no. of morning, engine can't start.

(i) $X \sim B(5, 0.4)$: $P(X \geq 2) = 1 - (P_0 + P_1)$

$\therefore P(X \geq 2) = 1 - [(0.6)^5 + 5(0.6)^4(0.4)] = 0.6630$

(ii) $X \sim B(50, 0.4) \rightarrow X \sim N(20, 12)$

$P(X < 25) \rightarrow P(X < 24.5) = P(Z < \frac{24.5-20}{\sqrt{12}})$

$= P(Z < 1.299) = 0.9030$ ✱

10.(a) $n = 100$, $\Sigma fx = 58950$

$\Sigma fx^2 = 40621025$

$\therefore \bar{x} = \frac{\Sigma fx}{100} = 589.5$ ✱

$s = \left[\frac{\Sigma fx^2}{100} - 589.5^2\right]^{1/2} = 242.28$ ✱

(b) $m = 599.5 + \left(\frac{50-40}{42}\right)(200) = 647.12$ ✱

mode $= 599.5 + \left(\frac{25}{25+24}\right)(200) = 701.54$ ✱

11. X = mass, $X \sim N(\mu, \sigma^2)$

$P(X > 85) = 0.05$: $\frac{85-\mu}{\sigma} = z_{0.05} \Rightarrow 85 - \mu = 1.6450$

$P(X < 25) = 0.25$: $\frac{25-\mu}{\sigma} = -z_{0.25} \Rightarrow 25 - \mu = -0.674$

$\therefore 60 = 2.319\sigma \Rightarrow \sigma = 25.87$ ✱

$\mu = 25 + 0.674\left[\frac{60}{2.319}\right] = 42.44$ ✱

Range $= (42.44 \pm z_{0.125}(25.87))$

$= (42.44 \pm 1.15(25.87)) = (12.69, 72.19)$ ✱

12.(a) $E[(X-\mu)^2] = \int_{-\infty}^{\infty} (x-\mu^2).f(x)dx$

$= \int_{-\infty}^{\infty} x^2.f(x) - 2\mu\int_{-\infty}^{\infty} x.f(x) + \mu^2\int_{-\infty}^{\infty} f(x)dx$

$= E(X^2) - 2\mu(\mu) + \mu^2(1)$

$= E(X^2) - [E(X)]^2$ ✱ $[\because \mu = E(X)]$

(b) (i) $\int_{-3}^{3} k(x+3)dx = 1 = \frac{k}{2}\left[(x+3)^2\right]_{-3}^{3}$

$\therefore \frac{k}{2}[36-0] = 1 \Rightarrow k = \frac{1}{18}$ ✱

(ii) $E(X) = \frac{1}{18}\int_{-3}^{3}(x^2+3x)dx = \frac{1}{18}\left[\frac{x^3}{3} + \frac{3x^2}{2}\right]_{-3}^{3}$

$= \frac{1}{18}\left[(9+9) + \frac{3}{2}[9-9]\right] = 1$ ✱

$E(X^2) = \frac{1}{18}\int_{-3}^{3}(x^3+3x^2)dx = \frac{1}{18}\left[\frac{x^4}{4} + x^3\right]_{-3}^{3}$

$= \frac{1}{18}\left[\frac{81-81}{4} + (27+27)\right] = 3$

$\therefore Var(X) = 3 - (1)^2 = 2$ ✱

(iii) $P(X \leq q) = \frac{1}{4} = \frac{1}{18}\int_{-3}^{q}(x+3)dx$

$\therefore \frac{q}{2} = \left[\frac{(x+3)^2}{2}\right]_{-3}^{q} \Rightarrow (q+3)^2 = q$

$\therefore q = 0$ ✱ $[\because -3 < q < 3]$

(iv) $Var(Y) = 1$: $a^2(2) = 1 \Rightarrow a = \frac{1}{\sqrt{2}}$ ✱ $(\because a > 0)$

$E(Y) = 0$: $a(1) + b = 0 \Rightarrow b = -\frac{1}{\sqrt{2}}$ ✱

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