MATHEMATICS T / S (PAPER 1)
SMJK CHUNG HWA, KOTA BAHRU (2005)

1. Prove that for any set of **A** and **B**, **A** ∩ (**B** – **C**) = (**A** ∩ **B**) – (**A** ∩ **C**). [6]

2. Solve the inequality $0 < \dfrac{x^2 - 5x + 4}{x^2 + 5x - 6} < 1$. [6]

3. Express $\dfrac{1}{(k+3)(k+1)}$ in partial fractions. [4]

   Hence, by multiplying with $\dfrac{1}{k+2}$, evaluate the value of $\displaystyle\sum_{k=1}^{20} \frac{1}{(k+3)(k+2)(k+1)}$. [3]

4. Find the inverse for $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 10 & 3 \\ 3 & 6 & 10 \end{pmatrix}$. Hence solve the simultaneous equations

   $x + 2y + 3z = 6$, $2x + 10y + 3z = 21$ and $3x + 6y + 10z = 20$. [8]

5. Function f is defined by $f(x) = 2x - |2x + 1| + |1 - x|$, $x \in \mathcal{R}$.

   (a) Show that $f(x) = \begin{cases} 3x + 2, & x < -\frac{1}{2} \\ -x, & -\frac{1}{2} \le x < 1 \\ x - 2, & x \ge 1 \end{cases}$. [3]

   (b) Sketch the graph of f. [2]

6. (a) Find $\displaystyle\int \frac{2x^2 + x - 5}{x - 4}\,dx$. [4]

   (b) By using suitable substitution, find $\displaystyle\int_1^4 \frac{1}{4 + \sqrt{x}}\,dx$. [5]

7. Given four numbers. The first three numbers form an arithmetic sequence while the last three numbers form a geometric sequence. If the sum of the first and the forth numbers is 16, and the sum of the second and the third numbers is 12, find the four possible numbers. [6]

8. Given that the complex number z and its conjugate z^* satisfy the equation $zz^* + 2iz = 12 + 6i$, find all the possible values of z.

   Hence, find the modulus and the argument for z. [7]

9. Given that $f(x) \equiv x^4 + ax^3 - 13x^2 + 27x - 12a$, where a is a constant, and $x + 3$ is a factor of $f(x)$.

   (a) Find the value of a. [2]

   (b) Show that 3 is the zero of $f(x)$. Hence, factorise $f(x)$ completely. [4]

   (c) Find the set of values of x for which

   (i) $f(x) > 0$ (ii) $f(|x|) > 0$. [3,4]

10. The tangent and normal at a point $P(t, e^{-\frac{t^2}{2}})$ on the curve $y = e^{-\frac{x^2}{2}}$ meet the x–axis at T and G respectively. Find the coordinates of T and G.

    N is the foot of the ordinate–x from P and NT.GN = e^{-t}, find the length of PN. [13]

11. Sketch the curve $xy = 6$ and the line $y = 9 - 3x$.

    Find the area bounded by the curve, the line and both the axes. [7]

    Find the volume of the solid generated when the above region is rotated about y–axis. [5]

12. The equation of a closed curve is $(x + y)^2 + 2(x - y)^2 = 24$.

    (a) Show that the gradient at the point (x, y) on the curve may be expressed in the form $\dfrac{3x - y}{x - 3y}$. [4]

    (b) Find the coordinates of all the points on the curve at which the tangent is parallel to the y–axis. [4]

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MATHEMATICS T (PAPER 2)
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1. Find the set of values of x in the range $0^\circ \le x \le 180^\circ$ that satisfies the equation $\sin 2x + \sin 3x + \sin 5x = 0$. [5]

2. Express $6 \sin x \cos x - 2\cos^2 x + 5$ in the form $A \sin 2x + B \cos 2x + C$, with A, B and C are constants to be determined.

   Hence, find the largest and smallest value for $\dfrac{1}{6\sin x \cos x - 2\cos^2 x + 5}$. [6]

3. In a certain type of chemical reaction, a substance A is continuously transformed into a substance B. Throughout the reaction, the sum of the masses of A and B remains constant and equal to m. Initially, only A is present and the mass of B present at time t after the start of the reaction is denoted by x. At any instant, the rate of increase of the mass of B is proportional to the mass of A.
   Determine the differential equation involving x and t. Solve the differential equation.

   Given also that $x = \dfrac{m}{2}$ when $t = \ln 2$, show that $x = m(1 - e^t)$. Hence,

   (a) find the value of x (in terms of m) when $t = 3 \ln 2$.

   (b) find the smallest value of t such that x exceeds 90% of m.

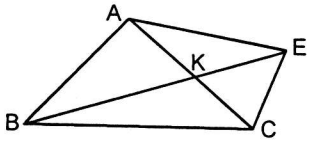
   (c) sketch the graph of x versus t. [12]

4. $\overrightarrow{OX}$ and $\overrightarrow{OY}$ are two vectors such that $\overrightarrow{OX} = \underline{a} - \underline{b}$ and $\overrightarrow{OY} = 2\underline{a} + 3\underline{b}$. If $\overrightarrow{OX}$ is perpendicular to $\overrightarrow{OY}$, and the magnitude of $\underline{a}$ and $\underline{b}$ are 7 units and 5 units respectively, find the angle between vectors $\underline{a}$ and $\underline{b}$. [5]

5. In the diagram, AB = AC and $\angle ABE = \angle ACE$.

   If $\angle BAC = \theta$, prove that $\angle AEC = 90^\circ + \dfrac{\theta}{2}$.

   If $\theta = 40^\circ$, $\angle EBC = 50^\circ$, find $\angle AKE$. [7]



6. (a) Prove that the diagonals of a rhombus meet each other at right angles. [4]

   (b) Find a vector which has magnitude 5 and is perpendicular to vector $3\underline{i} + \underline{j}$. [4]

7. (a) A fair die is tossed n times. If the probability a '6' does not occur at all is less than 0.01, what is the minimum value of n? [5]

   (b) A lot of 25 items is inspected following a two-stage inspection plan. A sample of 5 items is first drawn. If one or more is defective, the whole lot is rejected ; if all are good, a second sample of ten items is drawn from the remaining 20 items. The lot is rejected if any of the items in the second sample is defective, and accepted if all are good. Find the probability of accepting a lot that contains two defective items. [6]

8. Footballer X and footballer Y are kicking a ball to one another. The movement of the ball is always horizontal (on the grass) and in a straight line. At time $t = 0$, the ball is at the point with position vector $(6\underline{i} + 8\underline{j})$m relative to a fixed point O, with $\underline{i}$ and $\underline{j}$ as perpendicular unit vectors. X then kicks the ball with constant velocity $(\underline{i} - 2\underline{j})\text{ms}^{-1}$. At that moment Y is at a point with position vector $2\underline{i}$ m, and he is running with a constant velocity $(3\underline{i} + 2\underline{j})\text{ms}^{-1}$. Show that Y will intercept the ball and find the position vector of the point of interception. [8]

9. A car rental agent has only three cars each day which he can hires out. The number of demands for cars on each day is distributed as a Poisson distribution with mean 2. Calculate the percentage of days when

   (a) no cars are rented out. [3]

   (b) demand for a car cannot be met. [4]

10. The weights of fruits of an imported plant species grown in Malaysia is normally distributed with a variance of 0.04 kg^2. The mean is however, not known. The growers do know that 65% of the fruits weigh less than 0.5 kg each. Find the expected weight of a randomly chosen fruit. [5]

11. A continuous random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0 & , \quad x < 0 \\ \frac{1}{3}x & , 0 \le x < 2 \\ \frac{1}{12}(x+6) & , 2 \le x \le 6 \\ 1 & , 6 < x \end{cases}$$

   (a) Sketch a graph of $F(x)$ and determine the median for X. [4]

   (b) Determine the probability density function of X, $f(x)$, and sketch its graph. [3]

   (c) Show that $E(X) = 2$, and determine the variance of X. [5]

12. A random sample of people who own mobile phones was surveyed. The amount of time spent per day on their phones are as below.

Time spent per day (minutes)	0–	5–	10–	20–	30–	40–	70–
Relative frequency	0.12	0.21	0.32	0.19	0.10	0.06	0

   (a) Estimate the mean and standard deviation of the time spent per day on these mobile phones. [5]

   (b) By drawing a cumulative relative frequency curve, estimate the values of median and semi-interquartile range for the data. [5]

   (c) Draw a histogram to represent the data above. State the type of distribution and which measure of location is suitable to represent the above data. [4]

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1. $A\cap(B-C)=(B\cap C')\cap A = B\cap[C'\cap A]$

$= B\cap[(C'\cap A)\cup\emptyset] = B\cap[(C'\cap A)\cup(A'\cap A)]$

$= B\cap[(C'\cup A')\cap A] = B\cap[A\cap(C\cap A)']$

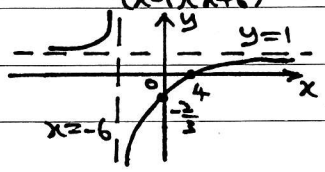
$= (B\cap A)\cap(A\cap C)' = (A\cap B)-(A\cap C)$ ✱

2. $0<\dfrac{x^2-5x+4}{x^2+5x-6}<1 \Rightarrow 0<\dfrac{(x-1)(x-4)}{(x-1)(x+6)}<1$

$\therefore\ 0<\dfrac{x-4}{x+6}<1$

$\therefore\ 0<1-\dfrac{10}{x+6}<1$

$\therefore\ \{x: 4<x\}$ ✱



3. Let $f(k)=\dfrac{1}{(k+1)(k+3)}=\dfrac{A}{k+1}+\dfrac{B}{k+3}$

$=\dfrac{A(k+3)+B(k+1)}{(k+1)(k+3)}$

$\therefore\ 1=A(k+3)+B(k+1)$

$k=-1$: $1=A(2)\Rightarrow A=\frac{1}{2}$

$k=-3$: $1=B(-2)\Rightarrow B=-\frac{1}{2}$

$\therefore\ f(k)=\frac{1}{2}\left[\dfrac{1}{k+1}-\dfrac{1}{k+3}\right]$ ✱

$f(k)\cdot\dfrac{1}{k+2}=\frac{1}{2}\left[\dfrac{1}{(k+1)(k+2)}-\dfrac{1}{(k+2)(k+3)}\right]$

$\therefore\ \sum_{k=1}^{20}\dfrac{1}{(k+1)(k+2)(k+3)}$

$=\frac{1}{2}\left[\left(\frac{1}{2(3)}+\frac{1}{3(4)}+\dots+\frac{1}{21(22)}\right)\right.$

$\left.-\left(\frac{1}{3(4)}+\dots+\frac{1}{21(22)}+\frac{1}{22(23)}\right)\right]$

$=\frac{1}{2}\left[\frac{1}{6}-\frac{1}{22(23)}\right]=\frac{125}{1518}$ ✱

4. $A=\begin{bmatrix}1&2&3\\3&10&3\\3&6&10\end{bmatrix}$: $(C_{ij})=\begin{bmatrix}82&-11&-18\\-2&1&0\\-24&3&6\end{bmatrix}$

$\therefore\ |A|=3(-18)+0+10(6)=6$

$\therefore\ A^{-1}=\frac{1}{6}(\text{adj }A)=\frac{1}{6}(C_{ij})^T$

$=\frac{1}{6}\begin{bmatrix}82&-2&-24\\-11&1&3\\-18&0&6\end{bmatrix}=\begin{bmatrix}41/3&-1/3&-4\\-11/6&1/6&1/2\\-3&0&1\end{bmatrix}$ ✱

$\left.\begin{array}{l}x+2y+3z=6\\2x+10y+3z=21\\3x+6y+10z=20\end{array}\right\}$ $A\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}6\\21\\20\end{pmatrix}$ — $\times A^{-1}$

$I\begin{pmatrix}x\\y\\z\end{pmatrix}=\frac{1}{6}\begin{pmatrix}82&-2&-24\\-11&1&3\\-18&0&6\end{pmatrix}\begin{pmatrix}6\\21\\20\end{pmatrix}$

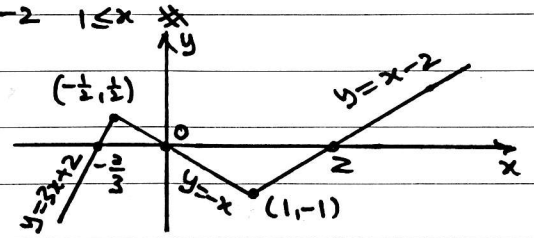
$\therefore\ \begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}-5\\2.5\\2\end{pmatrix}\Rightarrow x=-5, y=2.5, z=2$ ✱

5(a)

	$-\frac{1}{2}$		
$2x$	$2x$	$2x$	$2x$
$2x+1$	$-(2x+1)$	$-(2x+1)$	$-\lvert 2x+1\rvert$
$1-x$	$1-x$	$-(1-x)$	$\lvert 1-x\rvert$
$3x+2$	$-x$	$x-2$	$f(x)$

$\therefore\ f(x)=\begin{cases}3x+2 & x<\frac{1}{2}\\ -x & \frac{1}{2}\le x<1\\ x-2 & 1\le x\end{cases}$ ✱

(b)



6.(a) $\displaystyle\int\frac{2x^2+x-5}{x-4}\,dx$

$$\begin{array}{r|l} & 2x+9\\ x-4 & 2x^2+x-5\\ & 2x^2-8x\\ & 9x-5\\ & 9x-36\\ & 31\end{array}$$

$=\displaystyle\int\left[2x+9+\frac{31}{x-4}\right]dx$

$=x^2+9x+31\ln|x-4|+c$ ✱

(b) Let $u=4+\sqrt{x}$: $\dfrac{du}{dx}=\dfrac{1}{2\sqrt{x}}=\dfrac{1}{2(u-4)}$

$\displaystyle\int_1^4\frac{1}{4+\sqrt{x}}\,dx=\int_5^6\frac{1}{u}\cdot 2(u-4)\,du$

$=2\displaystyle\int_5^6\left(1-\frac{4}{u}\right)du=2\left[u-4\ln u\right]_5^6$

$=2\left[(6-4\ln 6)-(5-4\ln 5)\right]=2+8\ln\left(\frac{5}{6}\right)$ ✱

7. Let the no: $a, a+d, a+2d=(a+d)r, (a+d)r^2$

$T_2+T_3=12$: $(a+d)+(a+2d)=12=2a+3d$ — ①

and $(a+d)+(a+d)r=12=(a+d)(1+r)$ — ②

$T_1+T_4=16$: $S_4=28=3(a+d)+(a+d)r^2$

$\therefore\ 28=(a+d)(3+r^2)$ — ③

$\dfrac{③}{②}$: $\dfrac{7}{3}=\dfrac{3+r^2}{1+r}\Rightarrow 3r^2-7r+2=0$

$\therefore\ (3r-1)(r-2)=0\Rightarrow r=2$ or $\frac{1}{3}$

From ②: $(r=2)$ $3a+3d=12$ — ④

$\therefore$ ④ − ①: $a=0$ $\therefore\ d=4$

$\therefore$ 4 no. are $0, 4, 8, 16$ ✱

$r=\frac{1}{3}$: $12=(a+d)\left(\frac{4}{3}\right)\Rightarrow 2a+2d=18$ — ⑤

① − ⑤: $d=-6$ $\therefore\ a=15$

$\therefore$ 4 no. are $15, 9, 3, 1$ ✱

8. Let $z=a+bi$ $(a,b\in\mathbb{R})$

$z\bar{z}+2iz=12+6i$:

$(a+bi)(a-bi)+2i(a+bi)=12+6i$

$\therefore\ a^2+b^2-2b+2ai=12+6i$

[Im]: $2a=6\Rightarrow a=3$

[Re]: $9+b^2-2b=12\Rightarrow b^2-2b-3=0$

$\therefore\ (b-3)(b+1)=0\Rightarrow b=3$ or -1

$\therefore\ z=3+3i$ or $3-i$ ✱

$z=3(1+i)$: $|z|=3\sqrt{2}$ ✱, $\arg(z)=\tan^{-1}(1)=\frac{\pi}{4}$ ✱

$z=3-i$: $|z|=\sqrt{10}$ ✱

$\arg(z)=\tan^{-1}\left(\frac{-1}{3}\right)=-0.3218$ rad. ✱

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9. $(x+3)$ factor: $f(-3)=0$

(a) $81-27a-13(9)-81-12a=0 \Rightarrow a=-3$ ✱

(b) $f(3)=81-81-13(9)+81+36=0$

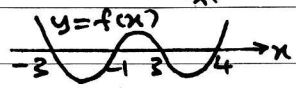
$\therefore$ 3 is a zero of $f(x)$ ✱

$\therefore f(x)=x^4-3x^3-13x^2+27x+36$

$=(x+3)(x-3)(x^2-3x-4)$

$=(x+3)(x-3)(x-4)(x+1)$ ✱

(c) (i) For $f(x)>0$:



$\therefore x \in (-\infty,-3)\cup(-1,3)\cup(4,\infty)$ ✱

(ii) For $f(|x|)>0$:

$\therefore |x| \in (-\infty,-3)\cup(-1,3)\cup(4,\infty)$

$\Rightarrow x \in (-\infty,-4)\cup(-3,3)\cup(4,\infty)$ ✱

10. $y=e^{-x^2/2} \Rightarrow \frac{dy}{dx}=-xe^{-x^2/2}$

$x=t$: $\frac{dy}{dx}=-te^{-t^2/2}$

Let $T(a,0)$, $G(b,0)$:

$m_{TP}=-te^{-t^2/2}=\frac{-e^{-t^2/2}}{a-t} \Rightarrow a=t+\frac{1}{t}$

$\therefore T(t+\frac{1}{t},0)$ ✱ $(t\neq 0)$

$m_{GP}=\frac{1}{t}e^{t^2/2}=\frac{-e^{-t^2/2}}{b-t}$

$\therefore b=t[1-e^{-t^2}]$ $\therefore G(t[1-e^{-t^2}],0)$ ✱

Let $N(t,0)$: $[NT.GN=e^{-t}]$

$\therefore |\frac{1}{t}|.|te^{-t^2}|=e^{-t} \Rightarrow e^{-t^2}=e^{-t}$

$\therefore -t^2=-t \Rightarrow t(t-1)=0$

$\therefore t=0$ or 1

[since $t\neq 0$]: $t=1$

$\therefore PN=e^{-\frac{1}{2}}=e^{-\frac{1}{2}}=\frac{1}{\sqrt{e}}$ ✱

1. $\sin 2x+\sin 3x+\sin 5x=0$

$\therefore 2\sin x\cos x+2\sin 4x\cos x=0$

$\therefore \cos x=0$ or $\sin x+\sin 4x=0$

$\therefore x=90°$ | $2\sin\frac{5x}{2}\cos\frac{3x}{2}=0$

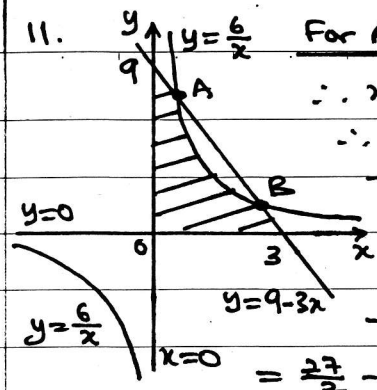
$\therefore \sin\frac{5x}{2}=0$ or $\cos\frac{3x}{2}=0$

$\therefore \frac{5x}{2}=0°,180°,360°$ | $\frac{3x}{2}=90°,270°$

$\therefore x=0°,72°,144°$ | $x=60°,180°$

$\therefore x=0°,60°,72°,90°,144°,180°$ ✱

11.



For A, B: $\frac{6}{x}=9-3x$

$\therefore x^2-3x+2=0$

$\therefore (x-1)(x-2)=0$

$\therefore x=1$ or 2

$\therefore A(1,6)$, $B(2,3)$

Area $=\frac{1}{2}(3)(9)-\int_1^2\left[9-3x-\frac{6}{x}\right]dx$

$=\frac{27}{2}-\left[9x-\frac{3x^2}{2}-6\ln x\right]_1^2$

$=\frac{27}{2}-\left[(18-6-6\ln 2)-(9-\frac{3}{2}-0)\right]$

$=9+6\ln 2$ ✱

$V_y=\frac{\pi}{3}(3)^2(9)+\pi\int_3^6\frac{36}{y^2}dy-\pi\int_3^6\frac{(9-y)^2}{9}dy$

$=27\pi+\pi\left[-\frac{36}{y}\right]_3^6+\frac{\pi}{27}\left[(9-y)^3\right]_3^6$

$=27\pi+\pi[-6+12]+\frac{\pi}{27}[27-216]=26\pi$ ✱

12. (a) $(x+y)^2+2(x-y)^2=24$ —$(\frac{d}{dx})$— ①

$2(x+y)(1+y')+4(x-y)(1-y')=0$

$\therefore x+y+(x+y)y'+2(x-y)-2(x-y)y'=0$

$\therefore [x+y-2x+2y]y'=y-3x$

$\therefore y'=\frac{3x-y}{x-3y}$ ✱ [Gradient at (x,y)]

(b) tangent // y-axis: (y' = undefined)

$\therefore x-3y=0 \Rightarrow x=3y$

from ①: $(4y)^2+2(2y)^2=24 \Rightarrow y=\pm 1$

$\therefore$ coordinates: $(3,1)$, $(-3,-1)$ ✱

2. Let $f(x)=6\sin x\cos x-2\cos^2 x+5$

$f(x)=3\sin 2x-[\cos 2x+1]+5$

$=3\sin 2x-\cos 2x+4$ ✱ $[=g(x)+4]$

Let $g(x)=R\sin(2x-\alpha)=R[\sin 2x\cos\alpha-\cos 2x\sin\alpha]$

$\therefore R\cos\alpha=3$, $R\sin\alpha=1$ | $\therefore R=\sqrt{3^2+1^2}=\sqrt{10}$

$\therefore \tan\alpha=\frac{1}{3} \Rightarrow \alpha=18.4°$

$\therefore \left[\frac{1}{f(x)}\right]_{max}=\frac{1}{\sqrt{10}(1)+4}=\frac{4+\sqrt{10}}{6}$ ✱

$\left[\frac{1}{f(x)}\right]_{min}=\frac{1}{\sqrt{10}(1)+4}=\frac{4-\sqrt{10}}{6}$ ✱

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3. $\frac{dx}{dt} \propto (m-x) \Rightarrow \frac{dx}{dt} = k(m-x)$ ✱

$\int_0^x \frac{dx}{m-x} = \int_0^t k\,dt \Rightarrow [\ln(m-x)]_0^x = [-kt]_0^t$

$\therefore \ln\left[\frac{m-x}{m}\right] = -kt \Rightarrow 1-\frac{x}{m} = e^{-kt}$

$\therefore x = m(1-e^{-kt})$ ✱

$t=\ln 2$: $\ln\left(\frac{m-\frac{1}{2}m}{m}\right) = -k\ln 2 = -\ln 2$

$\therefore k=1 \quad \therefore x = m(1-e^{-t})$ ✱

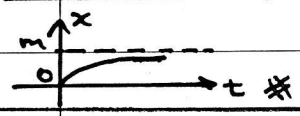
(a) $t=3\ln 2$: $x = m(1-e^{-3\ln 2}) = m(1-2^{-3})$

$= \frac{7}{8}m$ ✱

(b) For $x > 90\%m$: $t > -\ln\left(\frac{m-0.9m}{m}\right)$

$\therefore t > \ln 10$ ✱

(c)



✱

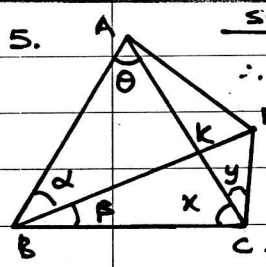
4. $\overrightarrow{OX} \perp \overrightarrow{OY}$: $(\underline{a}-\underline{b})\cdot(2\underline{a}+3\underline{b}) = 0$

$\therefore 2|\underline{a}|^2 - 3|\underline{b}|^2 + \underline{a}\cdot\underline{b} = 0$

$\therefore (7)(5)\cos\theta = 3(5)^2 - 2(7)^2 \Rightarrow \cos\theta = \frac{-23}{35}$

$\therefore \theta = 131.1°$ ✱ [angle between $\underline{a}$ and $\underline{b}$]

5.



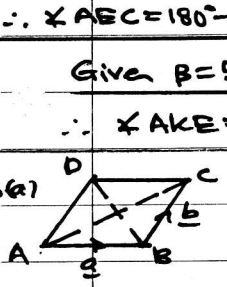
Statements	Reasons
$\alpha = y$	given
$\therefore$ ABCE are concyclic	Angles in the same segment
$\therefore \angle AEC = 180° - (\alpha+\beta)$	ABCD are concyclic
ΔABC isosceles with $x = \alpha+\beta$	Given AB = AC
$\therefore \theta + \alpha + \beta + x = 180°$	sum of angles in Δ
$\therefore \alpha+\beta = 90° - \frac{\theta}{2}$	

$\therefore \angle AEC = 180° - (90° - \frac{\theta}{2}) = 90 + \frac{\theta}{2}$ ✱

Given $\beta = 50°, \theta = 40°$: $\alpha = 90° - \frac{40°}{2} - 50° = 20°$

$\therefore \angle AKE = \alpha + \theta = 60°$ ✱ [exterior angle of ΔAKB]

6(a)



$\overrightarrow{AC}\cdot\overrightarrow{BD} = (\underline{a}+\underline{b})\cdot(\underline{b}-\underline{a})$

$= |\underline{b}|^2 + \underline{a}\cdot\underline{b} - \underline{b}\cdot\underline{a} - |\underline{a}|^2$

$= 0 \quad [\because \underline{a}\cdot\underline{b} = \underline{b}\cdot\underline{a},\ |\underline{a}| = |\underline{b}|]$

$\therefore$ Diagonals of rhombus meet at right angle. ✱

(b) Let $\begin{pmatrix}a\\b\end{pmatrix} \perp \begin{pmatrix}3\\1\end{pmatrix}$: $\begin{pmatrix}a\\b\end{pmatrix}\cdot\begin{pmatrix}3\\1\end{pmatrix} = 0 \Rightarrow 3a+b=0$

$\left|\begin{pmatrix}a\\b\end{pmatrix}\right| = 5$: $a^2+b^2 = 25 = a^2+9a^2 = 25$

$\therefore a = \pm\sqrt{2.5} \Rightarrow \begin{pmatrix}a\\b\end{pmatrix} = a\begin{pmatrix}1\\-3\end{pmatrix} = \sqrt{2.5}\begin{pmatrix}1\\-3\end{pmatrix}$ ✱

7.(a) Let X = no. of '6' occur, $X \sim B(n, \frac{1}{6})$

For $P(X=0) < 0.01$: $\left(\frac{5}{6}\right)^n < 0.01 \Rightarrow n\lg\left(\frac{5}{6}\right) < -2$

$\therefore n > 25.26 \Rightarrow n_{min} = 26$ [$\because n \in \mathbb{N}$] ✱

(b) Prob. $= \left(\frac{{}^{22}C_5}{{}^{25}C_5}\right)\left(\frac{{}^{18}C_{10}}{{}^{20}C_{10}}\right) = \frac{3}{20}$ ✱

8. $S_B = \begin{pmatrix}6\\8\end{pmatrix} + t\begin{pmatrix}1\\-2\end{pmatrix}$, $S_C = \begin{pmatrix}2\\0\end{pmatrix} + t\begin{pmatrix}3\\2\end{pmatrix}$

For interception: $S_B = S_C \Rightarrow \begin{pmatrix}6\\8\end{pmatrix} + t\begin{pmatrix}1\\-2\end{pmatrix} = \begin{pmatrix}2\\0\end{pmatrix} + t\begin{pmatrix}3\\2\end{pmatrix}$

$\therefore t\begin{pmatrix}2\\4\end{pmatrix} = \begin{pmatrix}4\\8\end{pmatrix} \Rightarrow t = 2$

$\therefore$ Chong will intercept the ball at $t = 2s$. ✱

Position vector of interception $= \begin{pmatrix}6\\8\end{pmatrix} + 2\begin{pmatrix}1\\-2\end{pmatrix} = \begin{pmatrix}8\\4\end{pmatrix}$ ✱

9. Let X = no. of demands for car, $X \sim P_o(2)$

(a) P(no car rented out) $= P(X=0) = e^{-2} = 0.1353$

(b) $P(X>3) = 1 - e^{-2}\left[1+2+\frac{2^2}{2!}+\frac{2^3}{3!}\right] = 0.1429$

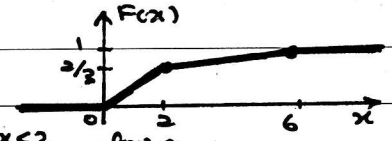
10. Let X = weight of fruit. $X \sim N(\mu, 0.04)$

$P(X<0.5) = 0.65 = P\left(Z < \frac{0.5-\mu}{0.2}\right)$

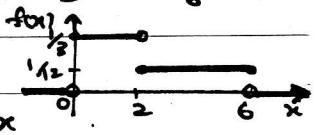
$\therefore \frac{0.5-\mu}{0.2} = z_{0.35} = 0.385 \Rightarrow \mu = 0.423\text{kg} = E(X)$ ✱

11.(a) $P(X \le m) = 0.5$

$\therefore \frac{1}{3}m = \frac{1}{2} \Rightarrow m = \frac{3}{2}$ ✱



(b) $f(x) = \frac{d}{dx}[F(x)] = \begin{cases} \frac{1}{3} & 0 \le x < 2 \\ \frac{1}{12} & 2 \le x \le 6 \\ 0 & \text{otherwise} \end{cases}$ ✱



(c) $E(X) = \int_0^2 \frac{1}{3}x\,dx + \int_2^6 \frac{1}{12}x\,dx$

$= \left[\frac{x^2}{6}\right]_0^2 + \left[\frac{x^2}{24}\right]_2^6 = \frac{4-0}{6} + \frac{36-4}{24} = 2$ ✱

$E(X^2) = \int_0^2 \frac{1}{3}x^2\,dx + \int_2^4 \frac{1}{12}x^2\,dx$

$= \left[\frac{1}{9}x^3\right]_0^2 + \left[\frac{1}{36}x^3\right]_2^6 = \frac{8-0}{9} + \frac{216-8}{36} = \frac{20}{3}$

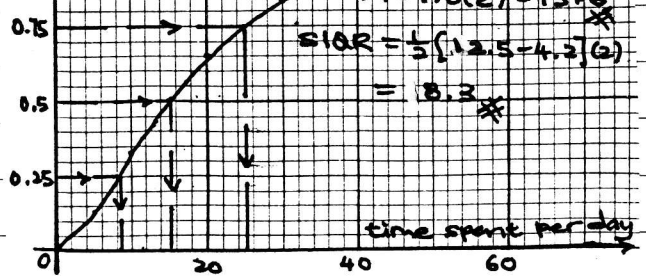
$\therefore Var(X) = \frac{20}{3} - 2^2 = \frac{8}{3}$ ✱

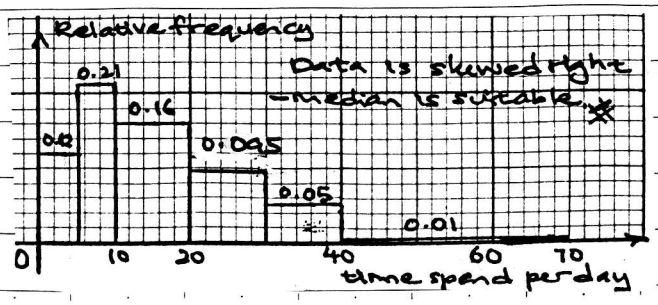
12.

midpoint	2.5	7.5	15	25	35	55
RF	0.12	0.21	0.32	0.19	0.10	0.06

(a) $\Sigma\left(\frac{f}{n}\right)x = 18.225 = \bar{x}$

$\Sigma\left(\frac{f}{n}\right)x^2 = 507.3125$: $s = \sqrt{507.3125 - 18.225^2} = 13.235$





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