JOHOR BAHRU (2005)
MATHEMATICS T (PAPER 1)

1. The function f is defined by $f: x \to \frac{1}{1+\sqrt{x}}$, $x \geq 0$.

   Find an expression for f^{-1} and state the domain and range of f^{-1}. [4]

2. Show that $\sum_{r=1}^{n} \frac{1}{4r^2-1} = \frac{n}{2n+1}$. Deduce the value of $\sum_{r=1}^{\infty} \frac{1}{4r^2-1}$. [5]

3. Given that $p(x) = 2x^3 + 5x^2 + ax - 6$, where a is a constant.
   If $(x + 2)$ is a factor of $p(x)$, find the value of a. [2]

   With this value of a, find all the zeros of $p(x)$. [3]

4. If $(x + iy)^2 = \frac{25}{3-4i}$, find all the real values of x and y. [6]

5. If $x^3 - 2xy^2 + y = 2$, find the values of y and $\frac{dy}{dx}$ when $x = -1$. [6]

6. The second and third term of a geometric progression are 24 and $12(b + 1)$ respectively. Find the value of b if the sum of the first three terms is 76. [6]

7. Find the equation of the line joining $A(2, 3)$ and $B(-4, 6)$. The line passing through $P(1, 11)$ and perpendicular to AB meets AB at Q. Find the equation of PQ. [4]

   PQ is produced to R such that $PQ = QR$. Find the coordinates of R and prove that the area of the quadrilateral $APBR$ is 45 units^2. [5]

8. (a) Show that $\int_1^2 \frac{6x+7}{(2x-1)(x+2)}\,dx = \ln 12$. [4]

   (b) By using the trapezium rule with 3 ordinates, obtain an approximation for the value of I, where $I = \int_1^2 \sqrt{x^3+1}\,dx$,

   giving your answer correct to three decimal places. [4]

   Draw a sketch of the curve $y = \sqrt{x^3+1}$ for $x \geq 0$, and hence, determine whether your approximation is greater than or smaller than the true value of I. [2]

9. Matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ -1 & -2 & 1 \end{pmatrix}$.

   (a) Show that $\mathbf{A}^2 = k\mathbf{A}$, where k is a constant. [2]

   (b) Show that **A** does not have an inverse. [3]

   (c) Verify that $(\mathbf{I} + \mathbf{A})^{-1} = \mathbf{I} - \frac{1}{7}\mathbf{A}$, where $\mathbf{I}$ is the 3 x 3 identity matrix. [5]

10. The parametric equations of a curve are given by $x = at^2$, $y = 2at$.
    Find the equation of the tangent to the curve at the point $(at^2, 2at)$. [4]

    $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ are two points on the curve, and O is the origin.
    If OP is perpendicular to OQ, show that $pq + 4 = 0$. [3]

    The tangents to the curve at P and Q meet at the point T. Find the coordinates of T.
    Hence, show that the locus of T is a straight line parallel to the y–axis. [4]

11. The region R in the first quadrant, is bounded by the line $y = 4 - x$ and the curve $y = \frac{4}{1+x}$ between their points of intersection.

    Find the coordinates of these points of intersection and sketch the region of R. [5]

    Show that the area of R is $\frac{1}{2}(15 - 16 \ln 2)$. [4]

    Find the volume of the solid form when R is rotated through one revolution about the x–axis. [4]

12. The function f is defined by $f(x) = 2 - \frac{1}{1+x^2}$.

    State all the real values of x for which $f(x)$ is defined. [1]

    State the equation of the asymptote and the axis of symmetry. [2]

    Find the coordinates and the nature of the stationary point, and show that there are points of inflexion at $x = \pm\frac{1}{\sqrt{3}}$. [7]

    Sketch the graph of $y = f(x)$. [3]

    Find the set of values of k such that the equation $2x^2 + 1 = k(x^2 + 1)$ has real solution. [2]

MATHEMATICS T (PAPER 2)

1. By using substitution $y = ux$, prove that the general solution for the differential equation $\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$ is $y^2 = x(x - A)$, where A is a constant. [5]

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2. The position vectors of the points A and B relative to the origin O are $-12\underline{i} + 5\underline{j}$ and $4\underline{i} + 3\underline{j}$ respectively. Find the angle between the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$. [3]

   Find the unit vector of the angle bisector of $\measuredangle AOB$. [3]

3. The town B is situated at a bearing of 120° and a distance of 320 km from town A. An aeroplane intends to fly from town A to town B. The wind is blowing at a speed of 50 kmh^{-1} from 330° during the whole flight. If the speed of the aeroplane in still air is 280^{-1} kmh ,find

   (a) the bearing of the course of the aeroplane, [4]

   (b) the flight duration from town A to town B. [3]

4. Express $3\cos x - 4\sin x$ in the form of $R\cos(x + \alpha)$, with $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [3]

   Hence, find the set of values of x, $0 \le x \le 2\pi$, for which satisfies the inequality $3\cos x - 4\sin x \ge -2.5$, giving your answer correct to three decimal places. [5]

5. The rate of change of the temperature of a cup of coffee is described by the differential equation $\frac{d\theta}{dt} = k(\theta - \theta_o)$, where θ is the temperature of the cup of coffee at time t, θ_o is the room temperature, and k is a constant.
   Explain why the value of k should be negative. [1]
   A cup of coffee at 88°C is left in a room that has a temperature of 28°C.
   The temperature of the cup of coffee after 10 minutes decreases to 68°C.
   Prove that $k = -\frac{1}{10}\ln\left(\frac{3}{2}\right)$. [5]
   When the temperature of the cup of coffee reaches 65°C, it is place in the manager's room that has a temperature of 15°C. Find the time required, from the moment the cup of coffee is in the manager's room until it is 35°C. [4]
   State your assumption. [1]

6. (a) $PQRS$ is a quadrilateral where $\measuredangle PQR$ and $\measuredangle QRS$ are obtuse and equal. If $PQ = 9$ cm, $QR = 6$ cm and $RS = 4$ cm,

   (i) prove that ΔPQR and ΔQRS are similar, [3]

   (ii) if T is the point of intersection of the diagonals, prove that QR is the tangent of the circle that passes through R, S and T. [4]

   (b) Vertices B and C of the triangle ABC lie on the circumference of a circle. AB and AC cut the circumference of the circle at P and Q respectively.

   Show that $\measuredangle CBP + \measuredangle CQP = 180^\circ$. [3]

   If $AB = AC$, show that BC is parallel to PQ. [3]

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7. The heights of 60 students is given as $\sum x = 10140$ cm and $\sum(x - 169)^2 = 10140\ \text{cm}^2$. Find the mean and standard deviation of the height of the students. [4]

8. The discrete random variable X has the probability distribution function given by $P(X = x) = kx(7 - x)$, for $x = 1, 2, 3, 4, 5$, where k is a constant.

   (a) Determine the value of k and tabulate the probability distribution of X. [2]

   (b) Find $P(|X - 3| > 1)$. [2]

9. A housing developer builds three types of house in an area, type A, B and C in the ratio 5 : 3 : 2. Each type of house has the larger size and the smaller size. The percentages of the larger houses among the types are 40% for A, 20% for B and 25% for C. It is found that the percentages of houses sold in he first week are 35% of A, 10% of B and 25% of C.

   (a) Find the percentage of the larger houses built by the developer. [2]

   (b) Find the percentage of houses sold in the first week are the larger houses. [5]

   (c) State whether the buyers prefer the larger or the smaller houses. [1]

10. T is a continuous random variable with probability density function

$$f(t) = \begin{cases} k & ,-2 < t \le -1 \\ kt & ,1 < t \le 3 \\ 0 & ,\text{otherwise} \end{cases}$$, where k is a constant.

   (a) Find the value of k. [3]

   (b) Find the cumulative distribution function of T. [4]

   (c) Deduce the value of h if $P(T \ge h) = 0.731$. [2]

11. The diameter of a type of screws has normal distribution with mean μ mm and standard deviation σ mm. Given that 10% of the screws have diameters more that 18.533 mm and 5% of the screws are with diameters not more than 17.362 mm.

   (a) Find the value of μ and σ, correct to 3 decimal places. [5]

   (b) Find the percentage of the screws, which have diameter between 17.58 mm and 17.88 mm. [3]

   (c) If 75% of the screws has diameter between 17.000 mm and k mm, find the value of k. [4]

12. The table shows the weights of 1000 students.

Weight (kg)	40 –	50 –	60 –	70 –	80 –	90 –	100 –
Relative frequency	1.000	0.954	0.800	0.567	0.284	0.084	0

   ["40 – " means 40 kg and above]

   (a) Find the mode of the weights of the students. [4]

   (b) Draw a cumulative frequency curve for the data. From the cumulative frequency curve, find the median and interquartile range of the weights of the students. [9]

1. Let $f^{-1}(x)=y$: $x=f(y)=\dfrac{1}{1+\sqrt{y}}$

$\therefore \sqrt{y}=\frac{1}{x}-1 \Rightarrow y=(\frac{1}{x}-1)^2$

$\therefore f^{-1}: x \to (\frac{1}{x}-1)^2,\ 0<x\le 1$

$D_{f^{-1}}: \{x: 0<x\le 1\}$, $R_{f^{-1}}: \{x: x\ge 0\}$

2. $\sum_{r=1}^{n} \frac{1}{4r^2-1} = \sum_{r=1}^{n} \frac{1}{2}\left[\frac{1}{2r-1}-\frac{1}{2r+1}\right]$

$=\frac{1}{2}\left[(1+\frac{1}{3}+\frac{1}{5}+\dots+\frac{1}{2n-1})-(\frac{1}{3}+\frac{1}{5}+\dots+\frac{1}{2n-1}+\frac{1}{2n+1})\right]$

$=\frac{1}{2}\left(1-\frac{1}{2n+1}\right)=\frac{n}{2n+1}$

$\sum_{r=1}^{\infty}\frac{1}{4r^2-1}=\lim_{n\to\infty}\left[\frac{1}{2}\left(1-\frac{1}{2n+1}\right)\right]=\frac{1}{2}(1-0)=\frac{1}{2}$

3. $(x+2)$ factor: $P(-2)=0$

$-16+20-2a-6=0 \Rightarrow a=-1$

$\therefore P(x)=2x^3+5x^2-x-6=(x+2)(2x^2+x-3)$

$=(x+2)(2x+3)(x-1)$

$\therefore$ zeros of $p(x)=-2, -\frac{3}{2}$ and 1

4. $(x+iy)^2=\frac{25}{3-4i} \Rightarrow x^2-y^2+2xyi=3+4i$

$\therefore x^2-y^2=3$ and $xy=2$

$\therefore x^2-(\frac{4}{x^2})=3 \Rightarrow x^4-3x^2-4=0$

$\therefore (x^2-4)(x^2+1)=0$

$\therefore x=\pm 2 \quad (\because x\in\mathbb{R})$

$\therefore x=2, y=1$ or $x=-2, y=-1$

5. $x^3-2xy^2+y=2$

$3x^2-2[x(2y\frac{dy}{dx})+y^2]+\frac{dy}{dx}=0$

$x=-1$: $-1+2y^2+y=2 \Rightarrow 2y^2+y-3=0$

$\therefore (2y+3)(y-1)=0 \Rightarrow y=-\frac{3}{2}$ or 1

$(-1,1)$: $3-2[-2\frac{dy}{dx}+1]+\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=-\frac{1}{5}$

$(-1,-\frac{3}{2})$: $3-2[3\frac{dy}{dx}+\frac{9}{4}]+\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=-\frac{3}{10}$

6. $ar=24$ and $r=\frac{12(b+1)}{24}=\frac{1}{2}(b+1)$

$a+24+12(b+1)=76$:

$24/\frac{(b+1)}{2}+12(b+1)=52 \quad$ (let $b+1=u$)

$\therefore 12+3u^2=13u \Rightarrow 3u^2-13u+12=0$

$\therefore (3u-4)(u-3)=0 \Rightarrow u=\frac{4}{3}$ or 3

$\therefore b+1=\frac{4}{3}$ or $3 \Rightarrow b=\frac{1}{3}$ or 2

7. AB: $\frac{y-3}{x-2}=\frac{6-3}{-4-2} \Rightarrow 2y+x=8$

PQ: $\frac{y-11}{x-1}=2 \Rightarrow y=2x+9$

For Q: $2(2x+9)+x=8 \Rightarrow x=-2 \quad \therefore Q(-2,5)$

$\therefore R=\left(\frac{2(-2)-1(1)}{2-1}, \frac{2(5)-1(11)}{2-1}\right)=(-5,-1)$

Area APBR $=\frac{1}{2}\begin{vmatrix} 2 & -4 & 1 & 2 \\ 3 & 6 & 11 & 3 \end{vmatrix} \times 2$

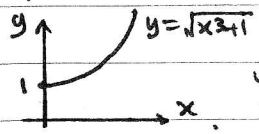
$=|(12-44+3)-(-12+6+22)|=|-45|=45$

8(a) $\int_1^2 \frac{6x+7}{(2x-1)(x+2)}dx=\int_1^2\left(\frac{1}{x+2}+\frac{4}{2x-1}\right)dx$

$=[\ln(x+2)+2\ln(2x-1)]_1^2=\ln\frac{4}{3}+2\ln 3$

$=\ln[\frac{4}{3}\times 3^2]=\ln 12$

(b) $I=\int_0^2\sqrt{x^3+1}\,dx=\frac{1}{2}(1)[1+3+2\sqrt{2}]=3.414$



since curve is concave upwards $\Rightarrow$ curve lower than trapeziums

$\therefore$ approximation > true value of I.

9.(a) $A^2=\begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ -1 & -2 & 1 \end{bmatrix}^2=\begin{bmatrix} 6 & 12 & -6 \\ 12 & 24 & -12 \\ -6 & -12 & 6 \end{bmatrix}=6A$

(b) $|A|=(1)(4-4)-2(2-2)-(1)(-4+4)=0$

$\therefore$ A is singular $\Rightarrow$ A does not have an inverse

(c) $(I+A)(I-\frac{1}{7}A)=\begin{bmatrix} 2 & 2 & -1 \\ 2 & 5 & -2 \\ -1 & -2 & 2 \end{bmatrix}(\frac{1}{7})\begin{bmatrix} 6 & -2 & 1 \\ -2 & 3 & 2 \\ 1 & 2 & 6 \end{bmatrix}$

$=\frac{1}{7}\begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}=I$

$\therefore (I+A)^{-1}=I-\frac{1}{7}A$

10. $x=at^2 \Rightarrow \frac{dx}{dt}=2at$, $y=2at \Rightarrow \frac{dy}{dt}=2a$

$\therefore \frac{dy}{dx}=\frac{2a}{2at}=\frac{1}{t}$

tangent at $(at^2, 2at)$: $\frac{y-2at}{x-at^2}=\frac{1}{t}$

$\therefore ty=x+at^2$

$OP \perp OQ$: $\left(\frac{2ap}{ap^2}\right)\left(\frac{2aq}{aq^2}\right)=-1=\frac{4}{pq}$

$\therefore pq+4=0$

tangent(P): $py=x+ap^2$ — ①

tangent(Q): $qy=x+aq^2$ — ②

① − ②: $(p-q)y=a(p-q)(p+q)$

$\therefore y=a(p+q) \quad \therefore x=apq=-4a$

$\therefore T(-4a, a(p+q))$

$\therefore$ locus of T: $x=-4a$ (a line parallel to the y-axis)

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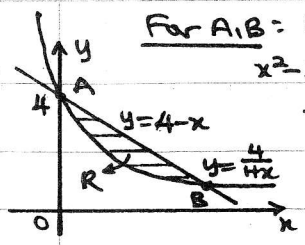
11. 

For A, B: $4-x=\frac{4}{1+x}$

$x^2-3x=0 \Rightarrow x(x-3)=0$

$\therefore A(0,4)$, $B(3,1)$

$\text{Area}=\int_0^3\left[4-x-\frac{4}{1+x}\right]dx$

$=\left[4x-\frac{x^2}{2}-4\ln(1+x)\right]_0^3$

$=12-\frac{9}{2}-4\ln 4-0=\frac{1}{2}[15-16\ln 2]$

$V_x=\pi\int_0^3\left[(4-x)^2-\frac{16}{(1+x)^2}\right]dx$

$=\pi\left[\frac{(4-x)^3}{-3}+\frac{16}{1+x}\right]_0^3$

$=\pi\left[\frac{1-64}{-3}+4-16\right]=9\pi$

12. $f(x)$ defined for all $x\in\mathbb{R}$.

asymptote: $y=2$

axis of symmetry: y-axis $(x=0)$

$f'(x)=\frac{2x}{(x^2+1)^2}$

$f''(x)=\frac{(x^2+1)^2(2)-2x(2x)(2)(x^2+1)}{(x^2+1)^4}$

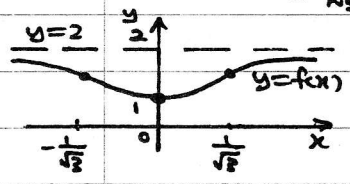
$=\frac{2(1-3x^2)}{(x^2+1)^3}$

For s.point: $f'(x)=0 \Rightarrow x=0$ $\therefore (0,1)$

$f''(0)=2\ (>0)$ $\therefore (0,1)$ min.

For inf. points: $f''(x)=0 \Rightarrow x^2=\frac{1}{3}$

$\therefore x=\pm\frac{1}{\sqrt{3}}$



$2x^2+1=k(x^2+1)$ — ①

$\therefore k=\frac{2x^2+1}{x^2+1}$

$\therefore k=2-\frac{1}{x^2+1}=y$

For ① to has real solution $\Rightarrow y=k$ and $y=f(x)$ have intersection points

$\therefore \{k: 1\le k<2\}$

1. $y=ux$: $\frac{dy}{dx}=u+x\frac{du}{dx}$

$\frac{dy}{dx}=\frac{x^2+y^2}{2xy}$: $u+x\frac{du}{dx}=\frac{x^2+u^2x^2}{2ux^2}=\frac{1+u^2}{2u}$

$\therefore \int\frac{2u}{1-u^2}du=\int\frac{dx}{x} \Rightarrow \ln(1-u^2)=-\ln x+c$

$\therefore \left(1-\frac{y^2}{x^2}\right)(x)=e^c=A \Rightarrow y^2=x(x-A)$

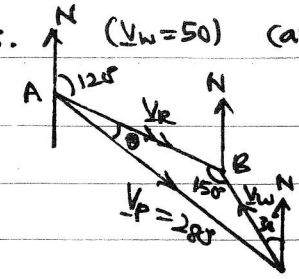
2. $\overrightarrow{OA}\cdot\overrightarrow{OB}=\begin{pmatrix}-12\\5\end{pmatrix}\cdot\begin{pmatrix}4\\3\end{pmatrix}=13(5)\cos\theta$

$\therefore \theta=\cos^{-1}\left[\frac{-48+15}{65}\right]=120.5°$ [Angle between OA and OB]

$\frac{\begin{pmatrix}a\\b\end{pmatrix}\cdot\begin{pmatrix}-12\\5\end{pmatrix}}{\sqrt{a^2+b^2}\,(13)}=\frac{\begin{pmatrix}a\\b\end{pmatrix}\cdot\begin{pmatrix}4\\3\end{pmatrix}}{\sqrt{a^2+b^2}\,(5)}$

$\therefore 5(-12a+5b)=13(4a+3b) \Rightarrow b=-8a$

$\therefore$ unit vector of angle bisector $=\frac{1}{\sqrt{65}}\begin{pmatrix}-1\\8\end{pmatrix}$

3. 

$(V_w=50)$ (a) $\frac{\sin\theta}{50}=\frac{\sin 150°}{280}$

$\therefore \theta=5.12°$

$\therefore$ bearing $=S\ 54.88°E$

(b) $\frac{V_R}{\sin 24.88°}=\frac{280°}{\sin 150°}$

$\therefore V_R=235.6$

$\therefore$ Duration $=\frac{320}{235.6}h=1h\ 21$ min.

4. $3\cos x-4\sin x=R\cos(x+\alpha)=f(x)$

$=R\cos x\cos\alpha-R\sin x\sin\alpha$

$\therefore R\cos\alpha=3$, $R\sin\alpha=4$

$\therefore \alpha=\tan^{-1}\left(\frac{4}{3}\right)=0.9273$ $\therefore R=5$

$\therefore f(x)=5\cos(x+0.9273)$

for $f(x)\ge -2.5$: $\cos(x+0.9273)\ge -0.5$

$\therefore 0.9273\le x+0.9273\le\frac{2\pi}{3}$ or $\frac{4\pi}{3}\le x+0.9273\le 2\pi+0.9273$

$\therefore \{x: 0\le x\le 1.167 \text{ or } 3.261\le x\le 2\pi\}$

5. $k<0$ because the temperature is decreasing per unit time (cooling)

$\frac{d\theta}{dt}=k(\theta-\theta_0)$: $\int_{88}^{\theta}\frac{d\theta}{\theta-28}=\int_0^t k\,dt$

$[\ln(\theta-28)]_{88}^{\theta}=k[t]_0^t \Rightarrow \ln\left(\frac{\theta-28}{60}\right)=kt$

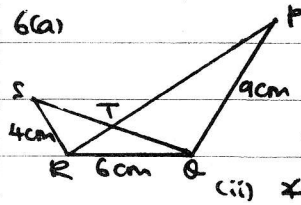
$t=10$: $\ln\left(\frac{40}{60}\right)=10k \Rightarrow k=-\frac{1}{10}\ln\left(\frac{3}{2}\right)$

$\theta_0=15°C$: $(t=0)$ $\ln\left(\frac{\theta-15}{65-15}\right)=-\frac{1}{10}\ln\left(\frac{3}{2}\right)\cdot t$

For $\theta=35$: $\ln\frac{20}{50}=-\frac{1}{10}\ln\left(\frac{3}{2}\right)\cdot t$

$\therefore t=\frac{-10\ln 0.4}{\ln 1.5}=22.60$ min.

Assumption: k is the same for both cases.

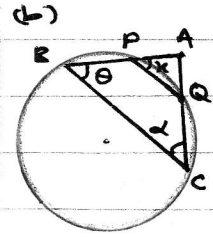
6(a) 

(i) $\angle PQR=\angle QRS$ and $\frac{PQ}{QR}=\frac{QR}{RS}=\frac{3}{2}$

$\therefore \Delta PQR\sim\Delta QRS$ — ①

(ii) $\angle TSR=\angle TRQ$ (by ①)

$\therefore$ QR is tangent to circle RST (a pair of equal alternate segment angles).

(b) 

Statements	Reasons
B C Q P are concyclic	B C Q P lies on the same circle
$\therefore \angle CBP+\angle CQP=180°$	opposite angles of cyclic quadrilateral are supplementary
$\theta=\alpha$	AB = AC (given) ΔABC is isosceles
$x=\alpha$	Exterior angle of cyclic BCQP.

$\therefore \theta=x \Rightarrow$ a pair of equal corresponding angle

$\therefore PQ\parallel BC$.

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7. $\bar{x} = \frac{10140}{60} = 169$

$\Sigma(x-169)^2 = 10140 = \Sigma(x-\bar{x})^2$

$\therefore\ s = \sqrt{\frac{10140}{60}} = 13$

8. (a) $\Sigma P(X=x)=1$:

$k[1(6)+2(5)+3(4)+4(3)+5(2)]=1 \Rightarrow k=\frac{1}{50}$

(b) $P(|X-3|>1) = P(X-3<-1)+P(X-3>1)$

$= P(X<2)+P(X>4) = P(X=1)+P(X=5)$

$= \frac{1}{50}[1(6)+5(2)] = 0.32$

9. (a) $P(L) = \frac{5}{10}(0.4)+\frac{3}{10}(0.2)+\frac{2}{10}(0.25)=0.21$

$\therefore$ Percentage $= 21\%$

(b) $P(LS) = \dfrac{0.5(0.4)0.35 + 0.3(0.2)0.1 + 0.2(0.25)(0.25)}{0.5(0.35)+0.3(0.1)+0.2(0.25)}$

$= \frac{0.0885}{0.255} = \frac{59}{170}$

$\therefore$ Percentage $= 34.71\%$

(c) Prefer smaller houses.

10. (a) $\int_{-2}^{-1} k\,dt + \int_1^3 kt\,dt = 1$

$k[t]_{-2}^{-1} + \frac{k}{2}[t^2]_1^3 = 1$

$k(-1+2)+\frac{k}{2}(9-1)=1 \Rightarrow k=\frac{1}{5}$

(b) $-2<t\le -1$: $P(T\le t) = \int_{-2}^{t}\frac{1}{5}dt = \frac{1}{5}[t+2]$

$-1<t\le 1$: $P(T\le t) = \frac{1}{5}[-1+2] = \frac{1}{5}$

$1<t\le 3$: $P(T\le t) = \frac{1}{5}+\int_1^t \frac{1}{5}t\,dt$

$= \frac{1}{5}+\frac{1}{10}(t^2-1) = \frac{1}{10}(t^2+1)$

$\therefore F(t) = P(T\le t) = \begin{cases} 0 & t\le -2 \\ \frac{1}{5}(t+2) & -2<t\le -1 \\ \frac{1}{5} & -1<t\le 1 \\ \frac{1}{10}(t^2+1) & 1<t\le 3 \\ 1 & 3<t \end{cases}$

(c) $P(T\ge h) = 0.731 \Rightarrow P(T<h) = 0.269$

$\therefore \frac{1}{10}(h^2+1) = 0.269$

$\therefore h = 1.3$ $(\because 1<h\le 3)$

11. Let X = diameter, $X \sim N(\mu, \sigma^2)$

(a) $P(X>18.533) = 0.1 = P(Z > \frac{18.533-\mu}{\sigma})$

$\therefore 18.533-\mu = \sigma . z_{0.1} = 1.282\sigma$ — ①

$P(X\le 17.362) = 0.05 = P(Z\le \frac{17.362-\mu}{\sigma})$

$\therefore 17.362-\mu = -z_{0.05}.\sigma = -1.645\sigma$ — ②

① − ②: $1.171 = 2.927\sigma \Rightarrow \sigma = 0.400$

$\therefore \mu = 17.362+1.645\left(\frac{1.171}{2.927}\right) = 18.020$

(b) $P(17.58 < X\ 17.88)$

$= P\left[\frac{17.58-18.020}{0.4} < Z < \frac{17.88-18.020}{0.4}\right]$

$= R(-1.1\ ) - R(-0.35\ )$

$= 0.8643 - 0.6368 = 0.2275$

$\therefore$ Percentage $= 22.75\%$

(c) $P(17<X<k) = 0.75$

$\therefore R\left(\frac{17-18.02}{0.4}\right) - R\left(\frac{k-18.02}{0.4}\right) = 0.75$

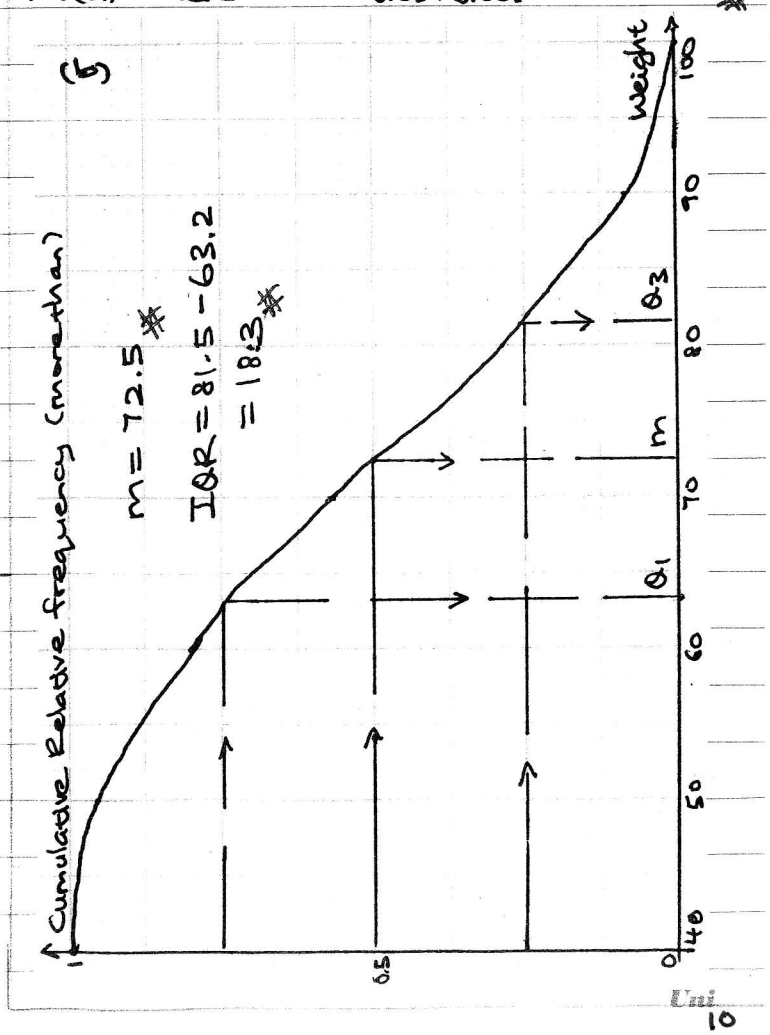
$\therefore 0.9946 - R\left[\frac{k-18.02}{0.4}\right] = 0.75$

$\therefore \frac{k-18.02}{0.4} = z_{0.2446} = 0.691$

$\therefore k = 18.296$

12. (a) Mode $= 70 + \frac{0.05}{0.05+0.083}(10) = 73.76$

(b)



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