DAERAH GOMBAK (2005)
MATHEMATICS T (PAPER 1)

1. Solve the equation $54^{x-1} = 7^{x+2}$, giving your answer correct to 3 decimal places. [4]

2. Given that $\frac{1}{3+i} + \frac{1}{a+bi} = \frac{3}{5}$. Find the values of a and b. [5]

3. Given that the matrix $\mathbf{A} = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{pmatrix}$. Find $\mathbf{A}$ and $adj(\mathbf{A})$.

   Show that $\mathbf{A}.\,adj(\mathbf{A}) = adj(\mathbf{A}).\mathbf{A} = |\mathbf{A}|.\mathbf{I}$, where $\mathbf{I}$ is the 3 x 3 identity matrix.
   Hence, find $\mathbf{A}^{-1}$. [8]

4. (a) Find $\int_1^{e^2} (\ln x)^2\, dx$. [6]

   (b) Use the trapezium with 7 ordinates to find an approximate value for $\int_1^4 \frac{1}{t}\, dt$, correct to 4 decimal places.

   Hence, estimate ln 4e correct to 3 decimal places. [6]

5. Polynomial $P(x)$ is given as $P(x) = ax^3 + bx^2 + cx + d$, where a, b, c and d are constants. Two of the roots of $P(x) = 0$ are $1 - 2i$ and 2. When $P(x)$ is divided by $(x + 1)$, the remainder is 48. Find the values of a, b, c and d.
   Find the set of values of x such that $P(x) > x - 2$. [12]

6. The area A bounded between the parts of the two curves $x^2 + y^2 = 4$ and $4x^2 + y^2 = 16$, for which $x \geq 0$, $y \geq 0$ is rotated completely about the x–axis.
   Find the volume of the solid formed. [6]

7. Sketch on the same diagram, the graphs of $9x^2 + 4y^2 = 36$ and $y = 2\sqrt{3x}$, giving the coordinates of the point of intersection. [6]

8. Functions f and g are defined as follows,

   $f : x \to \ln x, x > 0$ and $g : x \to \sqrt{x-1} - 1, x \in \mathbf{S}$.

   Find the largest possible set of $\mathbf{S}$ such that the composite function $f \circ g$ is defined.

   Hence, find $f \circ g$ and state its range. [6]

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9. (a) A geometric progression has non-zero first term a and common ratio r, where $0 < r < 1$. Given that the sum of the first 8 terms of the progression is equal to half the sum to infinity, find the value of r, correct to 3 decimal places. [3]
   If the 17th term of the progression is 10, find the value of a. [2]

   (b) An arithmetic progression has first term a and common difference 10. The sum of the first n terms of the progression is 10 000. Express a in terms of n and show that the nth term of the progression is $\frac{10\,000}{n} + 5(n - 1)$. [3]

   Given that the nth term is less than 500. Show that $n^2 - 101n + 2000 < 0$ and hence find the largest possible value of n. [4]

10. Expand $\frac{1 - x^2}{\sqrt[3]{1+3x}}$ in ascending powers of x up to and including terms in x^3. [4]

    State the values of x for which the series expansion is valid. [2]

11. (a) Given that $x^2 - 2xy + 2y^2 = 4$, find an expression for $\frac{dy}{dx}$ in terms of x and y. [3]

    (b) A curve is defined by the parametric equations $x = t^2$, $y = t^3$. Show that the equation of the tangent to the curve at the point $P(p^2, p^3)$ is $2y - 3px + p^3 = 0$. [3]

    Show that there one point on the curve at which the tangent passes through the point (–3, –5) and determine the coordinates of this point. [4]

12. Find the equation of the asymptotes of the curve $y = \frac{3 - 2x}{x - 2}$, and sketch the curve. [4]

    On the same diagram, sketch the graph of $y = 1 - e^{-2x}$. [2]

    Hence, state the number of real roots of the equation $(3x - 5)e^{2x} = x - 2$. [3]

    By taking $x = 1.7$ as the first approximation of the positive root, use the Newton-Raphson method to obtain an approximate value of the positive root, correct to 3 significant figures. [3]

MATHEMATICS T (PAPER 2)

1. Suppose A_o represents the amount of oil used at time $t = 0$, and the rate of decline of oil every year is k. Amount of oil A, used after time T, in years, is given by the formula $A = \int_0^T A_o e^{kt}\, dt$.

   In the year 2001 ($t = 0$) the usage of oil in the world is 30 billion barrels. The rate of decline of the world's oil is 10%. By assuming that this rate is fixed in future, find

   (a) the number of barrels of oil that will be used from the year 2001 until 2011. [3]

   (b) how long the worlds oil will last if it is known that there are 500 billion barrels of oil reserves in the year 2001. (Assume that no new oil fields are discovered). [4]

2. A pyramid has a horizontal square base of side a. Each sloping face is an isosceles triangle making an angle 30° with the base.

(i) Show that the height of the pyramid is $\frac{a}{2\sqrt{3}}$. [2]

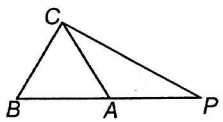
(ii) Show that the length of each sloping edge of a triangle face is $a\sqrt{\frac{7}{12}}$. [3]

(iii) Find, correct to the nearest degree, the angle between two adjacent triangular faces. [7]

3. When a man cycles due east at 2 ms^{-1}, the wind appears to come from the southeast. When he cycles due west at 1 ms^{-1}, the wind appears to come from the southwest. Sketch two separate velocity diagrams. [4]

Hence, find the actual velocity of the wind. [4]

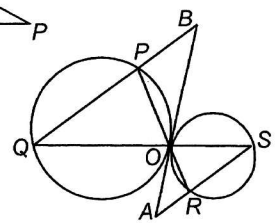
4. (a) Given that $AB = AC$ and BAP is a straight line. Prove that $PB > PC$. [4]



(b) In the diagram given, two circles touch at O, and AOB is a common tangent. BPQ, POR, ARS and QOS are straight lines. Prove that

(i) PQ is parallel to SR.

(ii) $\frac{BP}{AR} = \frac{BQ}{AS}$. [7]



5. Find $\cos 3x$ in terms of $\cos x$.

Hence, solve $\cos 3x + 2\cos x = 0$ for $0 \le x \le 2\pi$. [6]

6. At time 1500, the position vectors of two ship A and B relative to a fixed tracking station P are $(18\underline{i} + 6\underline{j})$ km and $(4\underline{i} + 10\underline{j})$ km respectively. A and B are moving with constants velocities $(\underline{i} + 4\underline{j})$ kmh^{-1} and $(3\underline{i} + 2\underline{j})$ kmh^{-1} respectively. Find the displacement of A relative to B at time t hours after 1500.

Hence, find the time taken to achieve the shortest distance between the ships. [6]

7. The events $\boldsymbol{A}$ and $\boldsymbol{B}$ are such that $P(\boldsymbol{A}) = 0.4$, $P(\boldsymbol{B} | \boldsymbol{A}) = 0.7$ and $P(\boldsymbol{A}' \cap \boldsymbol{B}) = 0.3$.

Find (a) $P(\boldsymbol{A} \cap \boldsymbol{B})$ (b) $P(\boldsymbol{B})$ (c) $P(\boldsymbol{A} \cup \boldsymbol{B})$.

State whether $\boldsymbol{A}$ and $\boldsymbol{B}$ are independent events. [8]

8. Find the median and the semi-interquartile range for the following data,

2 , 3 , 3 , 9 , 6 , 6 , 12 , 11 , 8 , 2 , 3 , 5 , 7 , 5 , 4 , 4 , 5 , 12 , 9 [5]

9. The number of cars which passes a road for 20 weeks are as follows,

67	58	68	58	49	57	68	35	41	79
79	49	58	69	51	48	57	69	49	43

(a) Draw a stem and leaf diagram for the above data.

(b) Calculate its mean, standard deviation and median.

(c) Draw a boxplot to represent the data.

(d) State the shape of the frequency distribution. Give a reason for your answer. [12]

10. The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} k\left(\frac{2}{x^2} - \frac{1}{2}\right) & , 1 \le x \le 2 \\ 0 & , \text{otherwise} \end{cases}$$, where k is a constant.

Show that $k = 2$. [3]

Find the cumulative distribution function of X, $F(x)$.

Hence, find the value of t for which $F(t) = \frac{2}{3}$. [6]

Find the mean of X, and show that the variance of X is approximately 0.0472. [5]

11. The discrete random variable X has probability distribution as follows,

$P(X = 1) = 0.2$ and $P(X = 2) = P(X = 3) = 0.4$

Y is another discrete random variable such that $X + Y = k$, where k is a constant. Given that $E(Y) = 1.8$.

(a) Find the values of k. [4]

(b) Find $P(2Y > 3)$. [2]

12. The number of television sets X, sold by an electrical shop per day follows a Poisson distribution with mean λ. Given that $E(X^2) = 2$.

(a) Show that $\lambda = 1$. [2]

(b) Find the probability that at least one television set is sold in a particular day. [3]

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1. $5^{4x-1} = 7^{x+2} \Rightarrow \frac{5^{4x}}{7^x} = 5(7^2)$

$\therefore x \lg\left(\frac{5^4}{7}\right) = \lg 5(7^2) \Rightarrow x = 1.225$ *

2. $\frac{1}{3+i} + \frac{1}{a+bi} = \frac{3}{5}$

$\therefore \frac{1}{a+bi} = \frac{3}{5} - \frac{1}{3+i} = \frac{4+3i}{5(3+i)} - \frac{4-3i}{4-3i}$

$\therefore a+bi = \frac{5(13-5i)}{25} = 3-i$

$\therefore a = 3$ and $b = -1$ *

3. $(C_{ij}) = \begin{bmatrix} 1 & -2 & -3 \\ -1 & -2 & -1 \\ -3 & 6 & 5 \end{bmatrix}$: $\text{adj}\,A = (C_{ij})^T = \begin{pmatrix} 1 & -1 & -3 \\ -2 & -2 & 6 \\ -3 & -1 & 5 \end{pmatrix}$ *

$|A| = 1(1) + (-2)(-2) + 3(-3) = -4$ *

$A.\text{adj}A = \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix} = -4I = |A|I$

$\text{adj}A.A = \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix} = -4I$

$\therefore A.\text{adj}A = \text{adj}A.A = |A|I$ *

$\therefore A^{-1} = \frac{1}{-4}\text{adj}A = \begin{bmatrix} -1/4 & 1/4 & 3/4 \\ 1/2 & 1/2 & -3/2 \\ 3/4 & 1/4 & -5/4 \end{bmatrix}$ *

4. (a) $\int_1^{e^2} (\ln x)^2 dx = \left[x(\ln x)^2\right]_1^{e^2} - \int_1^{e^2} x\left(\frac{2\ln x}{x}\right)dx$

$= e^2(2)^2 - 0 - 2\left[x\ln x\right]_1^{e^2} + \int_1^{e^2} 2\,dx$

$= 4e^2 - 2\left[e^2(2) - 0\right] + 2\left[x\right]_1^{e^2} = 2(e^2-1)$ *

(b) $\int_1^4 \frac{1}{t}dt = \frac{1}{2}\left(\frac{1}{2}\right)\left[1 + \frac{1}{4} + 2\left(\frac{1}{1.5} + \frac{1}{2} + \frac{1}{2.5} + \frac{1}{3} + \frac{1}{3.5}\right)\right] \approx 1.4054$ *

$\int_1^4 \frac{1}{t}dx = \left[\ln t\right]_1^4 = \ln 4$

$\therefore \ln 4e = \ln 4 + 1 = 1.4054 + 1 \approx 2.405$ *

5. Roots $= 1-2i, 2 \Rightarrow 1+2i$ is also a root

$\therefore P(x) = ax^2 + bx^2 + cx + d$

$\equiv a(x-2)(x^2-2x+5)$

$P(-1) = -48$: $-48 = a(-3)(1+2+5) \Rightarrow a = 2$ *

$\therefore P(x) = 2(x-2)(x^2-2x+5)$

$\therefore b = -8,\ c = 18,\ d = -20$ *

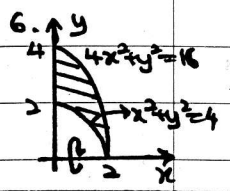
$P(x) > x-2$: $(x-2)\left[2(x^2-2x+5) - 1\right] > 0$

$\therefore (x-2)(2x^2-4x+9) > 0$

$\therefore (x-2)\left[2(x-1)^2+7\right] > 0$

$[\because 2(x-1)^2+7 > 0, \forall x \in \mathbb{R}]$: $x-2 > 0$

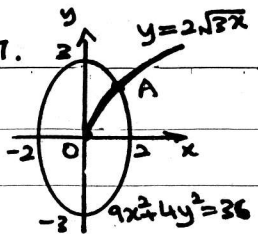
$\therefore \{x : x > 2\}$ *

6. 

$V_x = \pi\int_0^2 \left[(16-4x^2) - (4-x^2)\right]dx$

$= \pi\left[12x - x^3\right]_0^2$

$= \pi\left[24 - 8 - 0\right] = 16\pi$ unit^3 *

7. 

For A: $9x^2 + 4(12x) = 36$

$\therefore 3x^2 + 16x - 12 = 0$

$\therefore (3x-2)(x+6) = 0$

$[\because x_A > 0]$: $A\left(\frac{2}{3}, 2\sqrt{2}\right)$ *

8. fg is defined: S_{max} when $R_g = D_f$

$\therefore \sqrt{x-1} - 1 > 0 \Rightarrow \sqrt{x-1} > 1$

$\therefore x-1 > 1 \Rightarrow x > 2 \quad \therefore S = \{x : x > 2\}$ *

$fg(x) = f\left[\sqrt{x-1} - 1\right] = \ln\left[\sqrt{x-1} - 1\right]$

$\therefore fg : x \to \ln\left[\sqrt{x-1} - 1\right],\ x > 2$ *

$R_{fg} = R_f : \{x : x > 0\}$ *

9. (a) $S_8 = \frac{1}{2}S_\infty$: $\frac{a(1-r^8)}{1-r} = \frac{1}{2}\left(\frac{a}{1-r}\right)$

$\therefore r^8 = \frac{1}{2} \Rightarrow r = 0.917$ (3d.p) *

$T_{17} = 10$: $ar^{16} = 10 = a\left(\frac{1}{2}\right)^2 \Rightarrow a = 40$ *

(b) $S_n = 10000$: $\frac{n}{2}\left[2a + (n-1)(10)\right] = 10000$

$\therefore 2na = 20000 - 10(n-1)n$

$\therefore a = \frac{10000}{n} - 5(n-1)$ *

$T_n = a + (n-1)(10) = \frac{10000}{n} + 5(n-1)$ *

$T_n < 500$: $\frac{10000}{n} + 5(n-1) < 500$

$\therefore 10000 + 5n(n-1) < 500n$

$\therefore n^2 - 101n + 2000 < 0$ *

$\therefore \left(n - \frac{101}{2}\right)^2 - \left(\frac{101}{2}\right)^2 + 2000 < 0$

$\therefore (n - 50.5)^2 < 550.25$

$\therefore n_{max} < 50.5 + \sqrt{550.25} = 73.96$

$\therefore n_{max} = 73$ *

10. $\frac{1-x^2}{\sqrt[3]{1+3x}} = (1-x^2)\left[1 + \left(-\frac{1}{3}\right)(3x) + \frac{1}{2}\left(\frac{1}{3}\right)\left(-\frac{4}{3}\right)(3x)^2 + \frac{1}{3!}\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)(3x)^3 + \ldots\right]$

$= (1-x^2)\left[1 - x + 2x^2 - \frac{14}{3}x^3 + \ldots\right]$

$= 1 - x + x^2 - \frac{11}{3}x^3 + \ldots$ *

For VE: $|3x| < 1 \Rightarrow \{x : -\frac{1}{3} < x < \frac{1}{3}\}$ *

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11.(a) $x^2-2xy+2y^2=4$ —$(\frac{d}{dx})$

$\therefore 2x-2[x\frac{dy}{dx}+y]+4y\frac{dy}{dx}=0$

$\therefore \frac{dy}{dx}=\frac{x-y}{x-2y}$ ✱

(b) $x=t^2, y=t^3 \Rightarrow \frac{dx}{dt}=2t, \frac{dy}{dt}=3t^2$

$\therefore \frac{dy}{dx}=\frac{3t^2}{2t}=\frac{3}{2}t$

$P(p^2,p^3)$: $y-p^3=\frac{3}{2}p(x-p^2)$, (tangent)

$\therefore 2y-3px+p^3=0$ ✱

$(-3,-5)$: $-10+9p+p^3=0$

$\therefore (p-1)(p^2+p+10)=0$

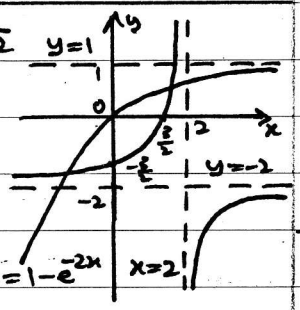
$\therefore (p-1)[(p+\frac{1}{2})^2+\frac{39}{4}]=0$

$[\because (p-\frac{1}{2})^2+\frac{39}{4}>0, \forall p\in\mathbb{R}]$ $\therefore p=1$

is the only real root. $\Rightarrow$ only 1 tangent passes through $(-3,-5)$. ✱

tangent point $=(1,1)$ ✱

12. $y=\frac{3-2x}{x-2}=-2-\frac{1}{x-2}$



asymptotes: $y=-2, x=2$

No. of real roots of $(3x-5)e^{2x}=x-2$

$\therefore \frac{3x-5}{x-2}=e^{-2x}$

$\therefore 1-e^{-2x}=-2-\frac{1}{x-2}$

$\therefore$ No of intersection points between the two curves $=2$ ✱

Let $f(x)=(3x-5)e^{2x}-x+2$

$f'(x)=(3x-5)(2e^{2x})+3e^{2x}-1=(6x-7)e^{2x}-1$

By NRM: $x_{n+1}=x_n-f(x_n)/f'(x_n)$

$x_0=1.7$: $x_1=1.7-\frac{3.2964}{94.885}=1.665\approx 1.67$

$x_2=1.665-\frac{0.19521}{82.536}=1.663\approx 1.66$

$x_3=1.663-\frac{0.02091}{81.868}=1.663\approx 1.66$

$\therefore$ root $=1.66$ (3s.f.) ✱

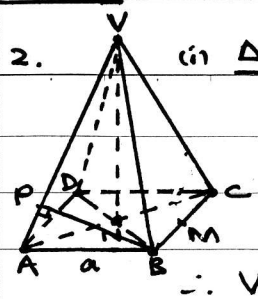
1.(a) $A_{10}=\int_0^{10}A_0e^{kt}\,dt$ $[A_0=30, k=0.1]$

$=\frac{30}{0.1}[e^{0.1t}]_0^{10}=515.48$ billion barrels ✱

(b) $500=\int_0^T 30e^{0.1t}\,dt=300[e^{0.1t}]_0^T$

$\therefore e^{0.1T}-1=\frac{5}{3}\Rightarrow T=10\ln\frac{8}{3}$ years ✱

2.



(i) ΔVNM: $\frac{h}{(\frac{a}{2})}=\tan 30^\circ$

$\therefore h=\frac{a}{2\sqrt{3}}$ ✱

(ii) $(\frac{a}{2})/VM=\cos 30^\circ$

$\therefore VM=\frac{a}{\sqrt{3}}$

$\therefore VB^2=(\frac{a}{\sqrt{3}})^2+(\frac{a}{2})^2\Rightarrow VB=\frac{a\sqrt{7}}{2\sqrt{3}}$ ✱

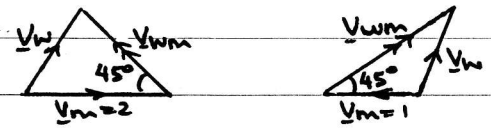
(iii) Area of $VAB=\frac{1}{2}(BP)(\frac{a\sqrt{7}}{2\sqrt{3}})=\frac{1}{2}(a)(\frac{a}{\sqrt{3}})$

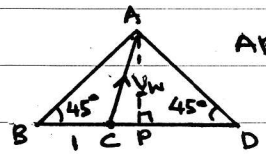
$\therefore BP=a\frac{2}{\sqrt{7}}$ $(=DP)$

$\therefore \cos\angle DPB=2(\frac{4a^2}{7})-(a^2+a^2)/2(\frac{2a}{\sqrt{7}})^2$

$\therefore \angle DPB=\cos^{-1}[-\frac{3}{4}]\approx 139^\circ$ ✱

3. $\underline{v}_{wm}=\underline{v}_w-\underline{v}_m\Rightarrow \underline{v}_w=\underline{v}_m+\underline{v}_{wm}$





$AB=AD$: $BP=PD=1.5$

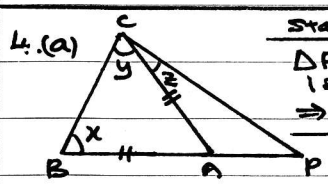
$AP=PD$ $(\because \angle ADP=45^\circ)$

$\therefore AC=\sqrt{0.5^2+1.5^2}=\frac{1}{2}\sqrt{10}$

$\angle ACP=\tan^{-1}(\frac{1.5}{0.5})=71.57^\circ$

$\therefore \underline{v}_w=\frac{1}{2}\sqrt{10}\ ms^{-1}$, to N 18.43°E ✱

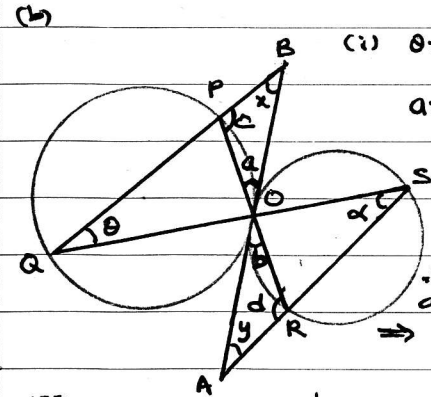
4.(a)



Statement	Reason
ΔABC is isosceles $\Rightarrow x=y$	$AB=AC$
$\angle BCP=z+y>x$	

$\therefore \angle BCP>\angle CBP\Rightarrow PB>PC$ ✱

(b)



(i) $\theta=a$	Angles of alternate segment
$a=b$	Opposite angles
$b=d$	Angles of alternate segment
$\therefore \theta=d$	
$\therefore$ A pair of equal alternate angles $\Rightarrow PQ\parallel SR$ ✱	

(ii) $x=y$	alternate angles, $PQ\parallel SR$
$c=d$	" " , $PQ\parallel SR$
$\therefore \Delta BPO\sim\Delta ARO$	
$\Rightarrow \frac{BP}{AR}=\frac{BO}{AO}$ — ①	
$\angle BOQ=\angle AOS$	
$\therefore \Delta BOQ\sim\Delta AOS$	
$\Rightarrow \frac{BQ}{AS}=\frac{BO}{AO}$ — ②	
$\therefore \frac{BP}{AR}=\frac{BQ}{AS}$ ✱	By ① and ②

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5. $\cos 3x = \cos(2x+x) = \cos 2x\cos x - \sin 2x \sin x$

$= [2c^2-1]c - 2sc(s) = 2c^3 - c - 2c(1-c^2)$

$= 4c^3 - 3c = 4\cos^3 x - 3\cos x$ *

$\cos 3x + 2\cos x = 0 \Rightarrow 4c^3 - 3c + 2c = 0$

$\therefore c(4c^2-1) = 0 \Rightarrow \cos x = 0$ or $\pm\frac{1}{2}$

$\therefore x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ *

6. $S_A = \binom{18}{6} + t\binom{1}{4}$, $S_B = \binom{4}{10} + t\binom{3}{2}$

$\therefore S_{AB} = S_A - S_B = \binom{14}{-4} + t\binom{-2}{2}$ *

For shortest distance : $S_{AB} \cdot \binom{-2}{2} = 0$

$-28 - 8 + t(4+4) = 0 \Rightarrow t = 4.5$ hr *

7. (a) $P(B|A) = \frac{P(A\cap B)}{P(A)} \Rightarrow 0.7 = \frac{P(A\cap B)}{0.4}$

$\therefore P(A\cap B) = 0.28$ *

(b) $P(B) = P(A\cap B) + P(A'\cap B)$

$= 0.28 + 0.3 = 0.58$ *

(c) $P(A\cup B) = P(A) + P(A'\cap B)$

$= 0.4 + 0.3 = 0.7$ *

$P(B) = 0.58 \ (\neq P(B|A))$

$\therefore$ A, B are not independent *

8. 2, 2, 3, 3, 3, 4, 4, 5, 5, 5, 6, 6, 7, 8, 9, 9, 11, 12, 12

$m = x_{10} = 5$ *

$\text{SIQR} = \frac{1}{2}(Q_3 - Q_1) = \frac{1}{2}[9-3] = 3$ *

9. (a)

3	5
4	1 3 8 9 9 9
5	1 7 7 8 8 8
6	7 8 8 9 9
7	9 9

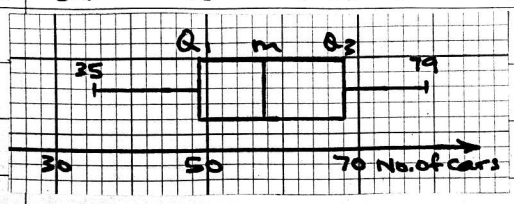
key: 3|5 means 35

(b) $\bar{x} = \frac{1152}{20} = 57.6$ *

$s = \left[\frac{69194}{20} - 57.6^2\right]^{1/2} = 11.91$ *

$m = \frac{1}{2}[57+58] = 57.5$ *

(c) $Q_1 = \frac{1}{2}[49+49] = 49$, $Q_3 = \frac{1}{2}[68+68] = 68$



(d) Distribution skewed right, because $m - Q_1 < Q_3 - m$ *

10. $\int_1^2 k\left(\frac{2}{x^2} - \frac{1}{2}\right)dx = 1 = k\left[-\frac{2}{x} - \frac{1}{2}x\right]_1^2$

$\therefore k\left[-1-1+2+\frac{1}{2}\right] = 1 \Rightarrow k = 2$ *

$1 \le t \le 2$: $P(X \le t) = \int_1^t \left(\frac{4}{x^2} - 1\right)dx$

$= \left[-\frac{4}{x} - x\right]_1^t = 5 - \frac{4}{t} - t$

$\therefore F(t) = \begin{cases} 0 & t < 1 \\ 5 - \frac{4}{t} - t & 1 \le t \le 2 \\ 1 & 2 < t \end{cases}$ *

$F(t) = \frac{2}{3}$: $5 - \frac{4}{t} - t = \frac{2}{3} \Rightarrow 3t^2 - 13t + 12 = 0$

$\therefore (3t-4)(t-3) = 0 \Rightarrow t = \frac{4}{3}$ ($\because 1 \le t \le 2$) *

$E(X) = \int_1^2 \left[\frac{4}{x} - x\right]dx = \left[4\ln x - \frac{1}{2}x^2\right]_1^2$

$= 4\ln 2 - \frac{1}{2}(4-1) = 4\ln 2 - \frac{3}{2}$ *

$E(X^2) = \int_1^2 (4 - x^2)dx = \left(4x - \frac{1}{3}x^2\right]_1^2$

$= 4(2-1) - \frac{1}{3}(8-1) = \frac{5}{3}$

$\therefore \text{Var}(X) = \frac{5}{3} - \left(4\ln 2 - \frac{3}{2}\right)^2 \approx 0.0472$ *

11. (a) $E(X) = 0.2(1) + 0.4(2+3) = 2.2$

$E(Y) = 1.8$: $1.8 = k - 2.2 \Rightarrow k = 4$ *

(b) $P(2Y > 3) = P(2(4-X) > 3)$

$= P(X < \frac{5}{2}) = 0.2 + 0.4 = 0.6$ *

12. $X \sim Po(\lambda)$

(a) $E(X^2) = 2$: $\text{Var}(X) + [E(X)]^2 = 2$

$\therefore \lambda + \lambda^2 = 2 \Rightarrow \lambda^2 + \lambda - 2 = 0$

$\therefore (\lambda - 1)(\lambda + 2) = 0 \Rightarrow \lambda = 1$ ($\because \lambda > 0$) *

(b) $P(X \ge 1) = 1 - e^{-1} = 0.6321$ *

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