KEDAH 2005
MATHEMATICS T / S (PAPER 1)

1. Using the laws of the algebra of set, prove that for any sets **A** and **B**,
$(\mathbf{A} \cap \mathbf{B}) \cup (\mathbf{A}' \cap \mathbf{B}') = (\mathbf{A} \cup \mathbf{B}') \cap (\mathbf{A}' \cup \mathbf{B})$. [4]

2. Find the equation of the circle with centre (2, –3) which touches the straight line $4x - 3y + 23 = 0$. [4]

3. If $(x + iy)^3 = x + iy$, find the real values of x and y. [6]

4. A geometric series has first term a and common ratio r, where $|r| < 1$. The sum to infinity of the series is 8. The sum to infinity of the series obtained by adding all the odd-numbered terms is 6. Find the values of r and a. [6]

5. The function f is defined by $f(x) = \begin{cases} \dfrac{x - |x|}{2} & , x < 0 \\ 1 + e^{1-x} & , x \geq 0 \end{cases}$.

   Find $\lim\limits_{x \to 0^+} f(x)$ and $\lim\limits_{x \to 0^-} f(x)$.

   Hence, determine whether f is continuous at $x = 0$. [6]

6. Expand $\sqrt{\dfrac{1+2x}{1-2x}}$ as a series of ascending powers of x up to and including the term in x^3. [4]

   By substituting $x = \dfrac{1}{50}$, find an approximation for $\sqrt{39}$ correct to 4 significant figures. [4]

7. The parametric equations of a curve are given by $x = t(t - 2)$ and $y = 2(t - 1)$. If P is a point on the curve and O is the origin, show that the equation of the locus of Q which divides OP internally in the ratio 1 : 2 is given by $9y^2 = 4(3x + 1)$. [3]
   If the normal at P passes through the point A(1, 8), find the coordinates of P. [6]

8. If α and β are real roots of the equation $x^2 - (k + 2)x + (k + 5) = 0$, find the ranges of values of k such that

   (i) both α and β are positive. [6]

   (ii) both α and β are positive and less than 7. [4]

9. Given that the matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{pmatrix}$ has an inverse of the form $A^{-1} = \begin{pmatrix} a & b & 0 \\ b & 0 & a \\ 0 & 2 & b \end{pmatrix}$.

   Find the values of a and b. [5]

   Hence, solve the following simultaneous equations,
   $x = \dfrac{5 - y - z}{2}$, $y = 2 - x - z$, $z = 3 - 2x - 2y$. [5]

10. Show that the equation $x^3 + 2x - 30 = 0$ has one and only one real root between 2 and 3. [4]

    By using the Newton-Raphson method, find the root correct to 3 decimal places. [7]

11. A solid of revolution is formed by rotating the region bounded by the y–axis, $y = 0$, $y = 1$ and the curve $y = e^{x^2} - 1$, about the y–axis.

    Show that the volume of the solid, V, is given by $V = \pi \int_0^1 \ln(y + 1)\, dy$. [2]

    Use the trapezium rule with 6 ordinates to find an approximation for V, correct to three significant figures. [5]

    Find the exact value of V. [5]

12. A curve has equation $y = f(x) = \dfrac{2 - x}{(x - 1)^2}$, $x \neq 1$.

    Find the equation of the asymptotes that are parallel to the coordinate axes and determine the coordinates of the turning point and its nature, if any. [7]

    Sketch, in separate diagrams, the curves $y = f(x)$ and $y = |f(x)|$. [5]

    State the number of real roots of the equation $|f(x)| = kx - k$. where $k < 0$. [2]

MATHEMATICS T (PAPER2)

1. Show that the equation $2 \sin\theta \cos\theta + 4 \cos^2\theta = 1$ may be written in the form $\tan^2\theta - 2 \tan\theta - 3 = 0$. [2]
   Hence, find the values of θ where $2 \sin\theta \cos\theta + 4 \cos^2\theta = 1$ for $0^\circ < \theta < 360^\circ$. [3]

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2. $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are two vectors such that $\overrightarrow{OA} = \underline{a} - 2\underline{b}$, $\overrightarrow{OB} = 2\underline{a} + \underline{b}$, and OA is perpendicular to OB.

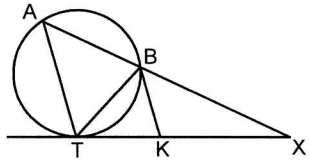
(i) Show that $\underline{a}.\underline{b} = (|\underline{a}|^2 - |\underline{b}|^2)$. [3]

(ii) If $\underline{a}$ has magnitude 4 units and $\underline{b}$ has magnitude 5 units, find $\cos\theta$, where θ is the angle between $\underline{a}$ and $\underline{b}$. [2]

3. A ship is moving on a constant speed of 10 kmh^{-1} in the direction of the unit vector $\underline{i}$. Initially, its position vector, relative to a fixed point, origin O, is $10(-\underline{i} + \underline{j})$, where $\underline{i}$ and $\underline{j}$ are perpendicular vectors of length 1 km.
Find its position vector relative to the origin at time t hours later. [1]

A second ship is moving with a constant speed of $10\sqrt{5}$ kmh^{-1} parallel to the vector $\underline{i} + 2\underline{j}$ and is initially at the origin. Show that the shortest distance between the ships is 10 km. [7]

4. In the diagram, ABX is a straight line. The line through B and parallel to AT meets the tangent XT at K. Prove that



(i) $XB^2 = XK.XT$ [4]

(ii) $XA : XB = TA^2 : TB^2$. [5]

5. A rectangular tank has a horizontal base with base area A m^2. Water is flowing into the tank at a constant rate p m^3s^{-1}, and flows out at a rate qx m^3s^{-1}, where x metres is the depth of the water in the tank at time t seconds and q is a constant. When the depth is 0.5 m, it remains at this constant value, show that

$\frac{dx}{dt} = k(2x - 1)$, where k is a positive constant. [4]

When $t = 0$, the depth of water in the tank is 0.75 m and is decreasing at a rate of 0.01 ms. Find the time at which the depth of water is 0.55 m. [6]

6. Prove the following identities,

(i) $2\cos\theta(\sin 3\theta - \sin\theta) = \sin 4\theta$. [3]

(ii) $4(\cos 4\theta + \cos 2\theta)\sin 3\theta \sin\theta = \cos 4\theta - \cos 8\theta$. [3]

Hence, without using tables or calculator, show that

(iii) $\sin 54^\circ - \sin 18^\circ = \frac{1}{2}$. [3]

(iv) $\sin 54^\circ \sin 18^\circ = \frac{1}{4}$. [3]

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7. The table shows 50 scores of a random sample in a computer game.

Score	Frequency
≥ 400 000	50
≥ 450 000	46
≥ 500 000	38
≥ 550 000	32
≥ 600 000	18
≥ 650 000	7
≥ 700 000	2

(i) Calculate the median score and the mode score. [4]

(ii) If the mean score of the data is 570 000, find the upper boundary of the last score class in this distribution. [2]

8. Given that $X \sim N(20, 25)$ and $Y \sim N(\mu, \sigma^2)$ where X and Y are independent random variables. If $P(X + Y > 75) = 0.06$ and $P(X - Y > 10) = 0.02$, calculate the values of μ and σ, correct to three significant figures. [6]

9. An unbiased cubical die has the number 1 on three faces, the number 2 on two faces and the number 3 on one face. The die is rolled twice and X denotes the total score. Find the probability distribution of X. [2]

Show that $E(X) = \frac{10}{3}$ and $Var(X) = \frac{10}{9}$. [5]

10. Four boys, Ali, Bala, Chong and Dave, and three girls, Esther, Fatimah and Goh, stand in a queue in random order. Find the probability that

(i) no two boys stand next to each other. [3]

(ii) all four boys stand next to each other. [3]

(iii) all four boys stand next to each other, given that at least two boys stand next to each other. [3]

11. The probability that a randomly chosen flight from KLIA is delayed by more than x hours is $\frac{1}{25}(x - 5)^2$, for $x \in \mathcal{R}$, $0 \le x \le 5$. No flight leaves early, and none is delayed for more than 5 hours.
The delay, in hours, for a randomly chosen flight is denoted by X.

(i) Find the median, m, of X, correct to three significant figures. [3]

(ii) Find the cumulative distribution function, F, of X. [3]

(iii) Find the probability density function, f, of X. [2]

(iv) Show that $E(X) = \frac{5}{3}$. [2]

12. On average, a rehabilitation centre receives one distress call every three days. A 'bad' week is a week in which 4 or more distress calls are received. Show that the probability that a week is a 'bad' week is 0.207, correct to three significant figures. [4]

Find the probability that

(i) in 10 randomly chosen weeks, at least 3 are 'bad' weeks. [4]

(ii) in 100 randomly chosen weeks, at least 30 are 'bad' weeks. [5]

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1. $(A\cap B)\cup(A'\cap B') = [A\cup(A'\cap B')]\cap[B\cup(A'\cap B')]$

$= [(A\cup A')\cap(A\cup B')]\cap[(B\cup A')\cap(B\cup B')]$

$= [E\cap(A\cup B')]\cap[(A'\cup B)\cap E]$

$= (A\cup B')\cap(A'\cup B)$ ※

2. Eq: $(x-2)^2+(y+3)^2 = \left[\frac{|4(2)-3(-3)+23|}{5}\right]^2$

$\therefore\ x^2-4x+4+y^2+6y+9 = 64$

$\therefore\ x^2+y^2-4x+6y-51=0$ ※

3. $(x+iy)^3 = x+iy \Rightarrow (x+iy)[(x+iy)^2-1]=0$

$\therefore\ x+iy=0$ or $x+iy=\pm1$

$\therefore\ x=0, y=0$ or $x=1, y=0$ or $x=-1, y=0$ ※

4. $S_\infty = 8$ and $S_{(odd)\infty} = 6$

$\frac{a}{1-r} = 8$ — ① $\quad \frac{a}{1-r^2} = 6$ — ②

$\frac{①}{②}$: $1+r = \frac{4}{3} \Rightarrow r = \frac{1}{3}$ ※

From ①: $a = 8(1-\frac{1}{3}) = \frac{16}{3}$ ※

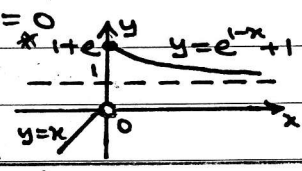
5. $f(x) = \begin{cases} \frac{x-|x|}{2}, & x<0 \\ e^{1-x}+1, & x\geq 0 \end{cases} = \begin{cases} x, & x<0 \\ e^{1-x}+1, & x\geq 0 \end{cases}$

$\lim_{x\to0^+} f(x) = \lim_{x\to0^+}[e^{1-x}+1] = e+1$ ※

$\lim_{x\to0^-} f(x) = \lim_{x\to0^-}[x] = 0$ ※

$\lim_{x\to0^+} f(x) \neq \lim_{x\to0^-} f(x)$

$\therefore$ f is not continuous at $x=0$ ※



6. $\sqrt{\frac{1+2x}{1-2x}} = (1+2x)^{\frac{1}{2}}(1-2x)^{-\frac{1}{2}}$

$= [1+\frac{1}{2}(2x)+\frac{\frac{1}{2}(-\frac{1}{2})}{2!}(2x)^2+\frac{1}{3!}(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})(2x)^3+\dots]$

$[1+(-\frac{1}{2})(-2x)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-2x)^2+\frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!}(-2x)^3+\dots]$

$= [1+x-\frac{1}{2}x^2+\frac{1}{2}x^3+\dots][1+x+\frac{3}{2}x^2+\frac{5}{2}x^3+\dots]$

$= 1+x[1+1]+x^2[\frac{3}{2}+1-\frac{1}{2}]+x^3[\frac{5}{2}+\frac{3}{2}-\frac{1}{2}+\frac{1}{2}]+\dots$

$= 1+2x+2x^2+4x^3+\dots = f(x)$ ※

$f(\frac{1}{50}) = \sqrt{\frac{26}{25}/\frac{24}{25}} = \sqrt{\frac{13}{12}} = \sqrt{\frac{39}{36}} = \frac{1}{6}\sqrt{39}$

$\therefore \sqrt{39} = 6[1+2(0.02)+2(0.02)^2+4(0.02)^3+\dots]$

$= 6.245$ (4s.f) ※

7. Let $Q(x,y) = (\frac{1}{3}t(t-2), \frac{2}{3}(t-1))$

$\therefore\ t = \frac{1}{2}(3y+2)$: $3x = \frac{1}{2}(3y+2)(\frac{1}{2})(3y-2)$

$\therefore\ 12x = 9y^2-4 \Rightarrow 9y^2 = 4(3x+1)$ [locus of Q] ※

$x = t(t-2), y = 2(t-1) \Rightarrow \frac{dx}{dt} = 2t-2, \frac{dy}{dt} = 2$

$\therefore \frac{dy}{dx} = \frac{2}{2t-2} = \frac{1}{t-1}$

$\therefore m_{AP} = -(t-1)$: $\frac{2(t-1)-8}{t(t-2)-1} = 1-t$

$\therefore\ 2t-10 = -t^3+3t^2-t-1$

$\therefore\ t^3-3t^2+3t-9=0 = t^2(t-3)+3(t-3)=0$

$\therefore (t-3)(t^2+3)=0 \Rightarrow t=3$ $[\because t\in\mathbb{R}]$

$\therefore P(3,4)$ ※

8. $x^2-(k+2)x+(k+5)=0$

(i) $\alpha,\beta>0$: $b^2-4ac\geq0$, $\alpha\beta>0$ and $\alpha+\beta>0$

$\therefore (k+2)^2-4(k+5)\geq0$ and $k+5>0$ and $k+2>0$

$\therefore k^2\geq16 \Rightarrow |k|\geq4$ and $k>-5$ and $k>-2$

$\therefore\ k\leq-4$ or $4\leq k$ and $k>-2$

$\therefore\ \{k: k\geq4\}$ — ① ※

(ii) $0<\alpha,\beta<7$: ① and $(\alpha-7)(\beta-7)>0$

$\therefore k\geq4$ and $\alpha\beta-7(\alpha+\beta)+49>0$

$\therefore (k+5)-7(k+2)+49>0$

$\therefore\ k<\frac{20}{3}$

$\therefore \{k: 4\leq k<\frac{20}{3}\}$ ※

9. $AA^{-1}=I$: $(1,2)$: $2b+0+2=0 \Rightarrow b=-1$ ※

$(1,3)$: $0+a+b=0 \Rightarrow a=1$ ※

$\left.\begin{array}{l} x=\frac{1}{2}(5-y-z) \\ y=2-x-z \\ z=3-2x-2y \end{array}\right\}$ $\left.\begin{array}{l} 2x+y+z=5 \\ x+y+z=2 \\ 2x+2y+z=3 \end{array}\right\}$ $A\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}5\\2\\3\end{pmatrix}$

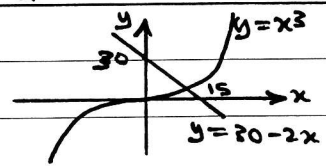
$\therefore \begin{pmatrix}x\\y\\z\end{pmatrix} = A^{-1}\begin{pmatrix}5\\2\\3\end{pmatrix} = \begin{pmatrix}1&-1&0\\-1&0&1\\0&2&-1\end{pmatrix}\begin{pmatrix}5\\2\\3\end{pmatrix} = \begin{pmatrix}3\\-2\\1\end{pmatrix}$

$\therefore\ x=3, y=-2, z=1$ ※

10. For $x^3+2x-30=f(x)=0$

$x^3 = 30-2x$

$\therefore y=x^3$ and $y=30-2x$



$\therefore$ Graphs intersects only once $\Rightarrow$ 1 real root only

$f(2)\cdot f(3) = (-18)(3)\ (<0) \Rightarrow$ root $\in(2,3)$

$\therefore f(x)=0$ has one and only one real root and the root lie between 2 and 3. ※

$f(x)=x^3+2x-30$ $\quad\therefore f'(x)=3x^2+2$

By NRM: $\alpha' = \alpha - \frac{f(\alpha)}{f'(\alpha)}$

Let $\alpha_0=2.9$: $\alpha_1 = 2.9-\frac{f(2.9)}{f'(2.9)}$

$\therefore \alpha_1 = 2.9-\frac{0.189}{27.23} = 2.8931 \approx 2.893$

$\therefore \alpha_2 = 2.8931-\frac{0.00153}{27.110} = 2.8930 \approx 2.893$

$\therefore$ root $= 2.893$ (3 d.p) ※

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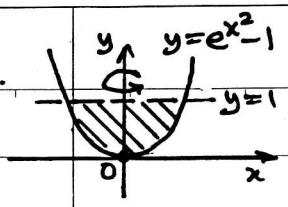
11. 

$y=e^{x^2}-1 \Rightarrow y+1=e^{x^2}$

$\therefore \ln(y+1)=x^2$

$V_y=\pi\int_0^1 x^2\,dy$

$\therefore V_y=\pi\int_0^1 \ln(y+1)\,dy$ ✱

By TR: $V_y=\pi\left(\frac{1}{2}\right)(0.2)\left[\ln1+\ln2+2(\ln1.2+\ln1.4+\ln1.6+\ln1.8)\right]=1.21$ (3 s.f.) ✱

$V_y=\pi\int_0^1 \ln(y+1)\,dy=\pi\int_1^2 \ln u\,du \quad [u=y+1]$

$=\pi\left[(u\ln u)_1^2-\int_1^2 u\left(\frac{1}{u}\right)du\right]$

$=\pi\left[2\ln2-0-(u)_1^2\right]$

$=\pi\left[2\ln2-(2-1)\right]=\pi[2\ln2-1]$ ✱

12. $y=f(x)=\dfrac{2-x}{(x-1)^2}$, $(x\neq1)$

asymptotes: $y=0$, $x=1$ ✱

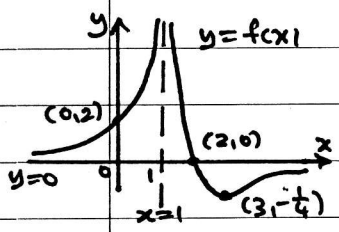
$y=\dfrac{2-(x-1+1)}{(x-1)^2}=\dfrac{1}{(x-1)^2}-\dfrac{1}{x-1}$

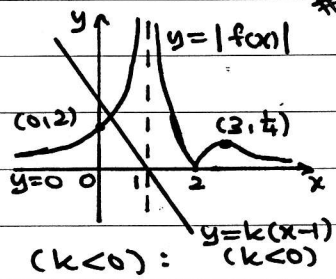
$\therefore \dfrac{dy}{dx}=\dfrac{-2}{(x-1)^3}+\dfrac{1}{(x-1)^2} \Rightarrow \dfrac{d^2y}{dx^2}=\dfrac{6}{(x-1)^4}-\dfrac{2}{(x-1)^3}$

For t.points: $\dfrac{dy}{dx}=0 \Rightarrow -2+(x-1)=0$

$\therefore x=3 \qquad \therefore P\left(3,-\frac{1}{4}\right)$

$x=3$: $\dfrac{d^2y}{dx^2}=\dfrac{6}{16}-\dfrac{2}{8}\ (>0) \quad \therefore P\left(3,-\frac{1}{4}\right)_{min}$ ✱





For $|f(x)|=kx-k$ $(k<0)$:

$y=|f(x)|$ and $y=k(x-1)$

$\therefore$ Exists only one intersection point

$\Rightarrow$ no. of real roots $=1$ ✱

1. $2\sin\cos\theta+4\cos^2\theta=1$ —— $(\div\cos^2\theta)$

$\therefore 2\tan\theta+4=\sec^2\theta=1+\tan^2\theta$

$\therefore \tan^2\theta-2\tan\theta-3=0$ ✱

$\therefore (t+1)(t-3)=0 \Rightarrow \tan\theta=-1$ or 3

$\therefore \theta=135°, 315°, 71.6°, 251.6°$ ✱

2. (i) $OA \perp OB$: $(\underline{a}-2\underline{b})\cdot(2\underline{a}+\underline{b})=0$

$\therefore 2|\underline{a}|^2+\underline{a}\cdot\underline{b}-4\underline{b}\cdot\underline{a}-2|\underline{b}|^2=0$

$[\because \underline{a}\cdot\underline{b}=\underline{b}\cdot\underline{a}]$: $\underline{a}\cdot\underline{b}=\frac{2}{3}\left[|\underline{a}|^2-|\underline{b}|^2\right]$ ✱

(ii) $|\underline{a}|=4, |\underline{b}|=5$: $4(5)\cos\theta=\frac{2}{3}[16-25]$

$\therefore \cos\theta=-0.3$ ✱

3. $\underline{s}_A=\underline{s}_{A(0)}+t.\underline{v}_A=10\begin{pmatrix}-1\\1\end{pmatrix}+t\left(10\begin{pmatrix}1\\0\end{pmatrix}\right)$

$\therefore \underline{s}_A=\begin{pmatrix}10t-10\\10\end{pmatrix}$ ✱

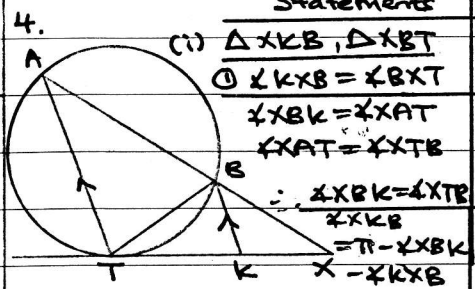
$\underline{v}_B=10\sqrt{5}\left[\frac{1}{\sqrt{5}}\begin{pmatrix}1\\2\end{pmatrix}\right]=\begin{pmatrix}10\\20\end{pmatrix}$

$\underline{s}_{AB}=\underline{s}_A-\underline{s}_B=\begin{pmatrix}10t-10\\10\end{pmatrix}-t\begin{pmatrix}10\\20\end{pmatrix}$

$=\begin{pmatrix}-10\\10-20t\end{pmatrix}=10\begin{pmatrix}-1\\1-2t\end{pmatrix}$

For $|\underline{s}_{AB}|_{min}$: $|\underline{s}_{AB}|_{min}=\left[10\sqrt{1+(1-2t)^2}\right]_{min}$

$\therefore |\underline{s}_{AB}|_{min}=10\sqrt{1}=10$ km ✱

4. 

Statements	Reasons
(i) $\Delta XKB, \Delta XBT$	
① $\angle KXB=\angle BXT$	common angle
$\angle XBK=\angle XAT$	Corresponding angles
$\angle XAT=\angle XTB$	Alternate segment angles (tangent)
$\therefore \angle XBK=\angle XTB$ — ②	
$\angle XKB=\pi-\angle XBK-\angle KXB$	sum of angles in ΔXKB.
$=\pi-\angle XTB-\angle BXT=\angle XBT$	sum of angles in ΔXBT

$\therefore \Delta XKB \sim \Delta XBT$

$\therefore \dfrac{XB}{XT}=\dfrac{XK}{XB} \Rightarrow XB^2=XK.XT$ ✱

Statements	Reasons
$\Delta XBK, \Delta XAT$	
$\angle XBK=\angle XAT$	corresponding angles
$\angle XKB=\angle XTA$	corresponding angles
$\angle KXB=\angle TXA$	common angle
$\therefore \Delta XBK \sim \Delta XAT$	

$\Rightarrow \Delta XAT \sim \Delta XTB \sim \Delta XBK$

$\therefore \dfrac{TA}{BT}=\dfrac{XT}{XB} \Rightarrow \dfrac{TA^2}{TB^2}=\dfrac{XT^2}{XB^2}=\dfrac{XT^2}{XK.XT}=\dfrac{XT}{XK}$

$\therefore \dfrac{TA^2}{TB^2}=\dfrac{XA}{XB}$ $(\because \Delta XAT \sim \Delta XBK)$

$\therefore XA:XB=TA^2:TB^2$ ✱

5. $\dfrac{dV}{dt}=p-qx$, $V=Ax \Rightarrow \dfrac{dV}{dt}=A\dfrac{dx}{dt}$

$\therefore A\dfrac{dx}{dt}=p-qx$

$x=0.5$: $\dfrac{dx}{dt}=0 \Rightarrow p-0.5q=0 \quad \therefore q=2p$

$\therefore \dfrac{dx}{dt}=\dfrac{p}{A}(1-2x)=-k(2x-1)$, $\left[k=\frac{p}{A}(>0)\right]$ ✱

$x=0.75$, $\dfrac{dx}{dt}=-0.01$: $-0.01=-k(1.5-1)$

$\therefore k=0.02=\dfrac{1}{50}$

$\dfrac{dx}{dt}=-\dfrac{1}{50}(2x-1) \Rightarrow \int_{0.75}^{0.55}\dfrac{2}{2x-1}\,dx=\int_0^t -\dfrac{1}{25}\,dt$

$\therefore [\ln(2x-1)]_{0.75}^{0.55}=-\dfrac{1}{25}[t]_0^t$

$\therefore \ln0.1-\ln0.5=-\dfrac{1}{25}[t-0]$

$\therefore t=25\ln5$

$\therefore$ when $x=0.55$, $t=25\ln5$ s ✱

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6.(i) $2\cos\theta[\sin 3\theta - \sin\theta]$

$= 2\sin 3\theta\cos\theta - 2\sin\theta\cos\theta$

$= [\sin(3\theta+\theta) + \sin(3\theta-\theta)] - \sin 2\theta$

$= \sin 4\theta$ ※

(ii) $4[\cos 4\theta + \cos 2\theta]\sin 3\theta\sin\theta$

$= 4[\cos 4\theta + \cos 2\theta](-\frac{1}{2})(\cos 4\theta - \cos 2\theta)$

$= -2[\cos^2 4\theta - \cos^2 2\theta]$

$= -[(\cos 8\theta + 1) - (\cos 4\theta + 1)] = \cos 4\theta - \cos 8\theta$ ※

(iii) $2\cos 18^\circ[\sin 3(18^\circ) - \sin 18] = \sin 4(18)$

$\therefore \sin 54^\circ - \sin 18^\circ = \dfrac{\sin 72^\circ}{2\sin(90^\circ - 18^\circ)} = \frac{1}{2}$ ※

(iv) $4[\cos 72^\circ + \cos 36^\circ]\sin 54^\circ \sin 18^\circ = \cos 72^\circ - \cos 144^\circ$

$\therefore \sin 54 \sin 18^\circ = \dfrac{\cos 72^\circ - (-\cos 36^\circ)}{4(\cos 72^\circ + \cos 36^\circ)} = \frac{1}{4}$ ※

7.

Score	f	Mid-point (x)
[400000, 450000)	4	425000
[450000, 500000)	8	475000
[500000, 550000)	6	525000
[550000, 600000)	14	575000
[600000, 650000)	11	625000
[650000, 700000)	5	675000
[700000, u)	2	k

(i) $m = x_{25} = 550000 + \left(\frac{25-18}{14}\right)(50000) = 575000$ ※

mode $= 550000 + \left(\frac{8}{8+3}\right)(50000) = 586364$ ※

(ii) $\bar{x} = 570000$: $\bar{x} = \dfrac{\Sigma fx}{50}$

$\therefore 50(570000) = 26950000 + 2k$

$\therefore k = 775000 = \frac{1}{2}[700000 + u]$

$\therefore u = 850000$ ※

8. $X+Y \sim N(\mu+20, \sigma^2+25)$

$X-Y \sim N(20-\mu, \sigma^2+25)$

$P(X+Y>75) = 0.06 = P\left(Z > \dfrac{75-\mu-20}{\sqrt{\sigma^2+25}}\right)$

$\therefore 1.555 = \dfrac{55-\mu}{\sqrt{\sigma^2+25}} \Rightarrow 1.555\sqrt{\sigma^2+25} = 55-\mu$ — ①

$P(X-Y>10) = 0.02 = P\left(Z > \dfrac{10-20+\mu}{\sqrt{\sigma^2+25}}\right)$

$\therefore 2.054\sqrt{\sigma^2+25} = \mu - 10$ — ②

① + ②: $3.609\sqrt{\sigma^2+25} = 45 \Rightarrow \sigma = 11.4$ ※

$\therefore \mu = 10 + 2.054\left[\frac{45}{3.609}\right] = 35.6$ ※

9. $X = 2, 3, 4, 5, 6$

$P_2 = P(1,1) = (\frac{2}{6})^2 = \frac{9}{36}$, $P_3 = P(1,2) = (\frac{2}{6})(\frac{2}{6})(2) = \frac{12}{36}$

$P_4 = P(1,3) + P(2,2) = (\frac{2}{6})(\frac{1}{6})(2) + (\frac{2}{6})^2 = \frac{10}{36}$

$P_5 = P(2,3) = (\frac{2}{6})(\frac{1}{6})(2) = \frac{4}{36}$

$P_6 = P(3,3) = (\frac{1}{6})^2 = \frac{1}{36}$ ※

$E(X) = \frac{1}{36}[2(9) + 3(12) + 4(10) + 5(4) + 6(1)]$

$= \frac{10}{3}$ ※

$E(X^2) = \frac{1}{36}[2^2(9) + 3^2(12) + 4^2(10) + 5^2(4) + 6^2(1)]$

$= \frac{110}{9}$

$\therefore Var(X) = \frac{110}{9} - (\frac{10}{3})^2 = \frac{10}{9}$ ※

10.(i) Prob $= \dfrac{3!(4!)}{7!} = \frac{1}{35}$ ※

(ii) Prob $= \dfrac{4!(4!)}{7!} = \frac{4}{35}$ ※

(iii) Prob $= \dfrac{P(\text{all boys together})}{1 - P(\text{no two boys together})}$

$= \frac{4}{35} / 1 - \frac{1}{35} = \frac{2}{17}$ ※

11. $P(X>x) = \frac{1}{25}(x-5)^2$, $0 \le x \le 5$

(i) $P(X>m) = \frac{1}{2}$: $\frac{1}{25}(m-5)^2 = \frac{1}{2}$

$\therefore (m-5)^2 = \frac{25}{2} \Rightarrow m = 5 - \sqrt{\frac{25}{2}}$ $(\because 0 \le m \le 5)$

$\therefore m = \frac{1}{2}[10 - \sqrt{25}]$ ※

(ii) For $0 \le x \le 5$: $P(X \le x) = 1 - P(X > x)$

$\therefore F(x) = P(X \le x) = \begin{cases} 0 & x<0 \\ 1 - \frac{1}{25}(x-5)^2 & 0 \le x \le 5 \\ 1 & 5 < x \end{cases}$ ※

(iii) $f(x) = \frac{d}{dx}[F(x)] = \begin{cases} \frac{2}{25}(5-x) & 0 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$ ※

(iv) $E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$

$= \frac{2}{25}\int_0^5 (5x - x^2)\,dx = \frac{2}{25}\left[\frac{5x^2}{2} - \frac{x^3}{3}\right]_0^5$

$= \frac{2}{25}\left[(\frac{5}{2}(25) - \frac{125}{3}) - 0\right] = \frac{5}{3}$ ※

12. Let X = no. of distress call in a week

$X \sim Po(\frac{7}{3})$

P(bad week) $= P(X \ge 4) = 1 - P(X<4)$

$= 1 - e^{-\frac{7}{3}}[1 + \frac{7}{3} + \frac{1}{2!}(\frac{7}{3})^2 + \frac{1}{3!}(\frac{7}{3})^3] = 0.207$ ※

(i) Y = no. of bad weeks, $Y \sim B(10, 0.207)$

$P(Y \ge 3) = 1 - [0.793^{10} + 10(0.793)^9(0.207) + 45(0.793)^8(0.207)^2] = 0.343$ ※

(ii) T = no. of bad weeks in 100 weeks

$T \sim B(100, 0.207) \Rightarrow T \sim N(20.7, 16.4151)$

$P(T \ge 30) \rightarrow P(T > 29.5)$

$= P\left(Z > \dfrac{29.5 - 20.7}{\sqrt{16.4151}}\right) = 0.0149$ ※

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