LEE HOCK LEONG
M.Sc. (Pure Maths) – CGPA 4
B.Sc with Ed. (1st Class Hons.)

SMJK KATHOLIK PJ (2006)
MATHEMATICS T (PAPER 1)

1. Sketch in the same diagram, the graphs of $y = |2x^2 - 1|$ and $y = \frac{4}{|x|} - 3$.

   Hence solve the inequality $|2x^2 - 1| > \frac{4}{|x|} - 3$. [6]

2. Given that Given that $zz^* = 5$ and $\frac{z}{z^*} = \frac{1}{5}(-3 + 4i)$.

   Express z in the form $a + bi$, where $a < b$. [4]

   Find the modulus and argument of z. Hence state the modulus and argument of z^*. [4]

3. The roots of the quadratic equation $x^2 - 28x + 16 = 0$ are denoted by α and β. Obtain a quadratic equation whose roots are $\sqrt{\alpha}$ and $\sqrt{\beta}$. [4]

4. The volume of a bowl, of depth h metres, cut from a sphere of radius a metres, where $h < a$, is $\frac{\pi}{3}h^2(3a - h)$ m^3. It is given that $h = \frac{1}{3}a$ and water is being poured into the bowl at a constant rate of $\frac{\pi}{24}a^3$m^3 per minute.

   Show that, when the depth of water in the bowl is x metres, $24\frac{dx}{dt} = \frac{a^3}{x(2a - x)}$. [4]

   Find the rate at which the depth of the water is rising when the bowl is full. [2]

5. Use the substitution $y = \sqrt{x}$, show that $\int_0^9 \sqrt{1+\sqrt{x}}\,dx = 2\int_0^3 y\sqrt{1+y}\,dy$. [2]

   Hence, find the exact value of $\int_0^9 \sqrt{1+\sqrt{x}}\,dx$. [4]

6. Find the determinant of $\begin{pmatrix} 2 & 3 & 2 \\ 2 & 3 & k^2-1 \\ 1 & 1 & 1 \end{pmatrix}$ in terms of k. [2]

   Hence, determine the value of k for which the following system of linear equation has no solution, $2x + 3y + 2z = 5$, $2x + 3y + (k^2 - 1)z = k + 1$ and $x + y + z = 2$ [2]

   If $k = 2$, solve the above system of equations. [6]

7. The equation of a curve is $y = \frac{2x^2 + 3x - 2}{x^2}$, $x \neq 0$.

   (a) Find the coordinates of the stationary point on the curve and determine its nature. [3]

   (b) Sketch the graph, indicating clearly all asymptotes, coordinates of all stationary points and any intersection with the axes. [5]

   (c) Using your sketch in (b), find the range of values of k for which the equation $(2 - k)x^2 + 3x - 2 = 0$ has two distinct real roots. [2]

8. The functions f and g are defined by

   $f : x \to \sin 2x,\ 0 \le x \le \pi$ and $g : x \to x^2 + 1,\ 0 < x \le 1$

   (a) Show, by means of a graphical argument, that f is not one to one. [3]

   (b) For the function $h : x \to \sin 2x,\ x \in S$, determine a maximal domain $S \subset [0, \frac{\pi}{2}]$, for h^{-1} to exist. Hence, find the corresponding h^{-1}. [2]

   (c) Explain why fg exists. Find fg and state its domain and range. [4]

9. The sequence of odd integers is grouped in brackets as follows :

   (1) , (3, 5) , (7, 9, 11) , (13, 15, 17, 19) , (21, . . .) , . . .

   such that each bracket contains one more integer than the preceding bracket.

   (a) Write down the total number of integers in the first n brackets and find the sum of all the integers in the first n brackets. [4]

   (b) Find the first integer in the nth bracket and find also the sum of the integers in the nth bracket. [5]

   (c) Hence show that $\sum_{r=1}^{n} r^3 = \left(\frac{n(n+1)}{2}\right)^2$. [2]

10. A straight line of gradient m passes through the point (2, 1) and cuts the x–axis and y–axis at A and B respectively. A point P lies on AB such that $AP : PB = 1 : 2$.

    (a) Show that as m varies, the locus of P is a curve with equation $3xy - x - 4y = 0$ [6]

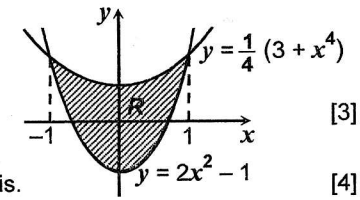
    (b) Find the perpendicular distance from the point (2, 1) to the tangent of the curve $3xy - x - 4y = 0$ at the origin. [5]

11. Find the equation of the normal to the parabola $y^2 = 4ax$ at the point $P(ap^2, 2ap)$. [3]

    If this normal meets the parabola again at the point $Q(aq^2, 2aq)$, show that $p^2 + pq + 2 = 0$. Deduce that q^2 cannot be less than 8. [4]

    If $\angle POQ = 90^\circ$, find the coordinates of P and Q. [4]

12. The diagram shows the shaded region R, bounded by the curves $y = \frac{1}{4}(3 + x^4)$ and $y = 2x^2 - 1$.



    (a) Find the area of R. [3]

    (b) Find the exact volume of the solid obtained when R is rotated through 2 right angles about the y–axis. [4]

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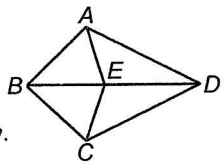
MATHEMATICS T (PAPER 2)

1. *ABCD* is a quadrilateral. Given that $AD = DC$ and $\angle ADE = \angle CDE$. Show $\angle BAE = \angle BCE$. [5]

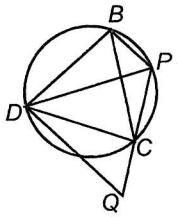


2. The variables x and y in the different equation $x^3 \frac{dy}{dx} = 2 \sec y$.

   Given that $x = 1$ when $y = \frac{\pi}{3}$. Find x^2 (in surd form) when $y = 0$. [6]

3. The respective scores, x for each of the 40 students of 5S1 for a mathematics test is summarized as $\sum x = 2380$ and $\sum x^2 = 149\,370$. The mean and standard deviation of scores, y for 45 students in 5S2 in a similar test are 52 and 8 respectively. Calculate to two decimal places, the mean and standard deviation of the marks for all 85 students from both classes. [6]

4. In the diagram, *BCD* is an equilateral triangle. *PCQ* is a straight line and $BP = CQ$. Show that



   (a) $\angle BPD = \angle CPD = 60^\circ$. [2]

   (b) triangles *PBD* and *QCD* are congruent. [2]

   (c) *DPQ* is an equilateral triangle. [3]

5. 7 out of 20 visitors to MPH Book Fair are expected to purchase books. Find the probability that at least two out of three visitors chosen at random would purchase books. 20000 visitors are expected for the second day of the Book Fair.

   By using a suitable approximation, calculate the probability that not more than 7050 of these visitors would purchase books.

6. If M and N are events where $P(M) = 1.2\,P(N)$, $P(M \cup N) = \frac{2}{3}$ and $P(N) = 2x$.

   Calculate x if

   (a) M and N are mutually exclusive. (b) N is a subset of M. [2,2]

   (c) M and N are independent. [3]

7. The mass of red dragon fruit produced by a farmer is normally distributed with a mean of 200 g and a standard deviation of 40 g. The mass of white dragon fruit produced by the farmer is normally distributed with a mean of 300 g and a standard deviation of 50 g.

   (a) Find the probability that the mass of a white dragon fruit, selected at random, is less than 250 g.

   Hence, find the probability that a white dragon fruit with mass less than 250 g has mass less than 200 g. [4]

   (b) If $Y = W - 2R$, where W represents the mass of a white dragon fruit and R, the mass of a red dragon fruit. Determine the mean and variance of Y.

   Assuming that Y is normally distributed, find the probability that the mass of white dragon fruit selected at random is more than twice the mass of a red dragon fruit selected at random. [4]

8. The time taken by each of the 40 runners in a 100 metre race in a school is as shown in the table :

Time (s)	Frequency
$11.5 < x \le 12.0$	2
$12.0 < x \le 12.5$	5
$12.5 < x \le 13.0$	9
$13.0 < x \le 13.5$	13
$13.5 < x \le 14.5$	8
$14.5 < x \le 16.0$	3

   (a) Plot a histogram to display the above data.

   (b) Use the histogram to estimate the median time.

   (c) If 10% of the runners were chosen to represent the school in an event find the maximum time taken by a chosen runner.

9. (a) By using the identity $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$, prove that
   $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$. [5]

   (b) Express $3\cos\theta - \sin\theta$ in the form of $R\cos(\theta + \alpha)$.

   Hence, find the maximum and minimum values of $3\cos\theta - \sin\theta$. [5]

10. X is a continuous random variable with probability density function given by

   $f(x) = \begin{cases} a(1+x) & ,2 < x < 5 \\ 0 & ,\text{otherwise} \end{cases}$, where a is a constant. Find the value of a. [3]

   (a) Find the value of b such that $P(X > b) = \frac{11}{27}$. [3]

   (b) Find the probabilities, $P(1 < X < 4)$ and $P(X \le 3)$. [4]

11. A town has N children. Hand Foot and Mouth (HFM) disease begin to spread among the children in this town. At time t, the number of children afflicted by HFM disease is n. The rate of increase in the number of children afflicted by the disease is directly proportional to the number of children not yet afflicted.

   Show that $\frac{dn}{dt} = k(N - n)$, where k is a positive constant and t is measured in weeks. [2]

   Hence, show that the no. of children, who have been afflicted with HFM disease at time t weeks since the disease started to spread in the town is given by $n = N - Ae^{-kt}$. [5]

   If initially, $N = 10000$ and $n = 600$, find A and hence, sketch the graph of n against t. [3]

   If $k = 1$, find the number of days that it takes to afflict 50% of the town's children. [2]

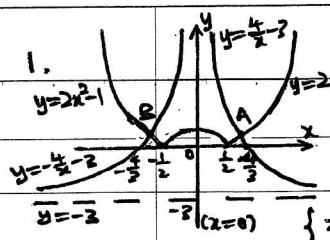
12. Two boats A and B travel with constant speed of $4\sqrt{2}$ km h^{-1} and 5 km h^{-1} in the direction north-east and S 36° E respectively.
   Find the magnitude and direction of the velocity of A relative to the velocity of B. [5]

   At 12 noon, B is 80 km to the north of A.

   (a) Determine the closest distance between these two boats.
   Find the distance travelled by each boat to reach this position. [6]

   (b) If the pilot of boat A intends to intercept boat B, find the direction that he must now steer boat A in order to intercept boat B. [3]

1.



For A: $2x^2-1=\frac{4}{x}-3$

$\therefore 2x^3+2x-4=0$

$2(x-1)(x^2+x+2)=0$

$\therefore x_A=1$

$\therefore x_B=-1$ [graphs are symmetrical]

For $|2x^2-1|>\frac{4}{|x|}-3$:

$\{x: x<-1 \text{ or } 1<x\}$ ✱

2. $z=a+bi$: $\frac{z}{z^*}=\frac{1}{5}[-3+4i]$

$\therefore 5(a+bi)=(-3+4i)(a-bi)$

$\therefore 5a+5bi=(-3a+4b)+i(4a+3b)$

$\therefore b=2a$

$zz^*=5$: $a^2+b^2=5 \Rightarrow 5a^2=5$

$\therefore a=\pm1 \Rightarrow z=1+2i$ ✱ $(\because a<b)$

$|z|=\sqrt{5}$ ✱, $\arg(z)=\tan^{-1}\left(\frac{2}{1}\right)=1.107$

$|z^*|=|z|=\sqrt{5}$ ✱, $\arg(z^*)=-\arg(z)=-1.107$ ✱

3. $x^2-28x+16=0$ (roots $=\alpha,\beta$)

$\therefore \alpha+\beta=28$, $\alpha\beta=16$

Roots $=\sqrt{\alpha},\sqrt{\beta}$: Product $=\sqrt{\alpha\beta}=4$

Sum $=\sqrt{\alpha}+\sqrt{\beta}=[\alpha+\beta+2\sqrt{\alpha\beta}]^{1/2}$

$=(28+2(4))^{1/2}=6$

$\therefore$ Eq: $y^2-6y+4=0$

4. $V=\frac{\pi}{3}h^2(3a-h)=\frac{\pi}{3}(3ah^2-h^3)$

when $h=x$: $V=\frac{\pi}{3}[3ax^2-x^3]$

$\frac{dV}{dt}=\frac{\pi}{3}[6ax-3x^2]\frac{dx}{dt}=\frac{\pi a^3}{24}$

$\therefore 24\frac{dx}{dt}=\frac{a^3}{x(2a-x)}$ ✱

$x=\frac{1}{3}a$: $\frac{dx}{dt}=a^3/\frac{1}{3}a\left(\frac{5a}{3}\right)=\frac{3}{40}a$ m per min ✱

5. $y=\sqrt{x}$: $y^2=x \Rightarrow 2yy'=1$

$\int_0^9\sqrt{1+\sqrt{x}}\,dx=\int_0^3\sqrt{1+y}\,(2y\,dy)=2\int_0^3 y\sqrt{1+y}\,dy$ ✱

$\therefore \int_0^9\sqrt{1+\sqrt{x}}\,dx=2\int_0^3(1+y-1)\sqrt{1+y}\,dy$

$=\int_0^3\left[2(1+y)^{3/2}-2\sqrt{1+y}\right]dy$

$=\left[\frac{4}{5}(1+y)^{5/2}-\frac{4}{3}(1+y)^{3/2}\right]_0^3$

$=\left[\frac{4}{5}(32)-\frac{4}{3}(8)\right]-\left[\frac{4}{5}-\frac{4}{3}\right]=\frac{232}{15}$

6. Det $=2(2-3)+(1-k^2)(2-3)+1(6-6)=k^2-3$ ✱

For No Solution: Det $=0 \Rightarrow k=\pm\sqrt{3}$ ✱

$k=2$: $\text{adj}=\begin{pmatrix}0&1&-1\\-1&0&1\\3&-2&0\end{pmatrix}^T=\begin{pmatrix}0&-1&3\\1&0&-2\\-1&1&0\end{pmatrix}$

$A\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}5\\3\\2\end{pmatrix}\Rightarrow\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}0&-1&3\\1&0&-2\\-1&1&0\end{pmatrix}\begin{pmatrix}5\\3\\2\end{pmatrix}=\begin{pmatrix}3\\1\\-2\end{pmatrix}$

$\therefore x=3, y=1, z=-2$ ✱

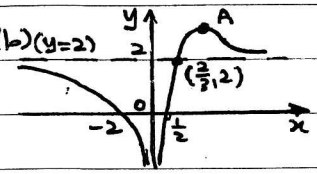
7. $y=\frac{2x^2+3x-2}{x^2}=\frac{(2x-1)(x+2)}{x^2}=2+\frac{3}{x}-\frac{2}{x^2}$

(a) $y'=-\frac{3}{x^2}+\frac{4}{x^3}=\frac{4-3x}{x^3}$, $y''=\frac{6}{x^3}-\frac{12}{x^4}=\frac{6x-12}{x^4}$

$y'=0$: $x=\frac{4}{3} \Rightarrow$ stat. point $=A\left(\frac{4}{3},\frac{25}{8}\right)$ ✱

$y''=-4/\left(\frac{4}{3}\right)^4\ (<0) \Rightarrow$ A is a max point ✱

(b) (y=2)



(c) $(2-k)x^2+3x-2=0$

$\therefore 2x^2+3x-2=kx^2$

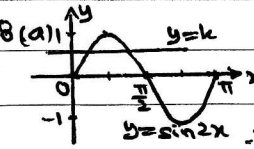
$\therefore \frac{2x^2+3x-2}{x^2}=k$

For 2 real and distinct roots:

$\Rightarrow y=k$ cut the graphs at two diff. points.

$\therefore \{k: k<2 \text{ or } 2<k<\frac{25}{8}\}$ ✱

8 (a)



$y=k$ cuts $y=f(x)$ at 2 points

f is many-1 $\Rightarrow$ f is not 1-1 ✱

(b) $S=\{x: 0\le x\le\frac{\pi}{4}\}$ ✱

$\therefore h^{-1}: x\to\frac{1}{2}\sin^{-1}(x)$, $0\le x\le1$ ✱

(c) R_g: $(1,2] \Rightarrow R_g\subset D_f$

$\therefore fg$ exists. ✱

$fg: x\to\sin 2(x^2+1)$, $0<x\le1$ ✱

D_{fg}: $\{x: 0<x\le1\}$, R_{fg}: $\{y: \sin4\le x<\sin2\}$ ✱

9. (a) Let U_n = no. of terms in the nth bracket

U_n: 1, 2, 3, 4 ... $\Rightarrow U_n=n$

$\sum_{r=1}^{n}U_r=\frac{n}{2}(n+1)$ ✱ $=N$

$S=\frac{N}{2}[2(1)+(N-1)(2)]=N^2=\frac{n^2}{4}(n+1)^2$ ✱

(b) $T(n)=1+\left[\left(\frac{n-1}{2}\right)(n)-1\right](2)+2=n^2-n+1$ ✱

$S(n)=\frac{n}{2}[2(n^2-n+1)+(n-1)(2)]=n^3$ ✱

(c) $\sum_{r=1}^{n}S(r)=S$: $\sum_{r=1}^{n}r^3=\left[\frac{n}{2}(n+1)\right]^2$ ✱

10. (a) AB: $\frac{y-1}{x-2}=m \Rightarrow A\left(2-\frac{1}{m},0\right)$, $B(0,1-2m)$

Let $P(x,y)$: $x=\frac{2}{3}\left(2-\frac{1}{m}\right)$ and $y=\frac{1}{3}(1-2m)$

$\therefore 3x-4=-\frac{2}{m}$ and $3y-1=-2m$

$\therefore (3x-4)(3y-1)=4 \Rightarrow 9xy-12y-3x=0$

$\therefore 3xy-x-4y=0$ [locus of P] ✱

(b) $3xy'+3y-1-4y'=0$

$O(0,0)$: $y'=-\frac{1}{4} \Rightarrow$ tangent: $y=-\frac{1}{4}x$

$d=\frac{|4(1)+(2)|}{\sqrt{16+1}}=\frac{6}{\sqrt{17}}$ ✱

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11. $y^2=4ax$: $2yy'=4a \Rightarrow y'=\frac{2a}{y}$

$P(ap^2,2ap)$: $y'=\frac{2a}{2ap}=\frac{1}{p}$

$\therefore$ Normal (P): $\frac{y-2ap}{x-ap^2}=-p$

$\therefore y+px=2ap+ap^3$ *

$m_{PQ}=m_{N(P)}$: $\frac{2ap-2aq}{ap^2-aq^2}=-p$

$\therefore \frac{2}{p+q}=-p \Rightarrow p^2+pq+2=0$ *

$[\because p,q\in\mathbb{R}]$: $B^2-4AC\geq 0 \Rightarrow q^2-8\geq 0$

$\therefore q^2\geq 8 \Rightarrow q^2\not< 8$ *

$\angle POQ=90^\circ$: $m_{OP}\cdot m_{OQ}=-1=\left(\frac{2ap}{ap^2}\right)\left(\frac{2aq}{aq^2}\right)$

$\therefore pq=-4 \Rightarrow -2-p^2=-4$

$\therefore p=\pm\sqrt{2}$

$p=\sqrt{2}$: $P(2a,2\sqrt{2}a)$, $Q(8a,-4\sqrt{2}a)$ *

$p=-\sqrt{2}$: $P(2a,-2\sqrt{2}a)$, $Q(8a,4\sqrt{2}a)$ *

12 (a) Area $=2\int_0^1\left[\frac{1}{4}(3+x^4)-(2x^2-1)\right]dx$

$=\left[\frac{3}{2}x+\frac{1}{10}x^5-\frac{4}{3}x^3+2x\right]_0^1$

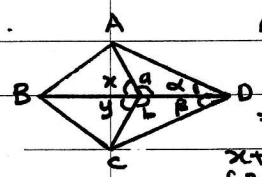
$=\left(\frac{3}{2}+\frac{1}{10}-\frac{4}{3}+2\right)-(0)=\frac{34}{15}$ *

(b) $V_y=\pi\int_{-1}^{1}\frac{1}{2}(y+1)dy-\pi\int_{3/4}^{1}\sqrt{4y-3}\,dy$

$=\frac{\pi}{4}\left[(y+1)^2\right]_{-1}^{1}-\frac{\pi}{6}\left[(4y-3)^{3/2}\right]_{3/4}^{1}$

$=\frac{\pi}{4}[4-0]-\frac{\pi}{6}[1-0]=\frac{5\pi}{6}$ *

1.



$AD=CD$, $\alpha=\beta$ (given)

$DE=DE$ (common side)

$\therefore \Delta ADE\equiv\Delta CDE$ (by SAS)

$\Rightarrow a=b$ and $AE=CE$

$x+a=180^\circ$ and $y+b=180^\circ$ [supplementary angles]

$\therefore x=y$, $BE=BE$ (common side

$\therefore \Delta ABE\equiv\Delta CBE$ (by SAS)

$\Rightarrow \angle BAE=\angle BCE$ *

2. $x^3\frac{dy}{dx}=2\sec y \Rightarrow \int_{\pi/3}^{0}\cos y\,dy=\int_1^x\frac{2}{x^3}dx$

$\therefore [\sin y]_{\pi/3}^{0}=\left[\frac{-1}{x^2}\right]_1^x \Rightarrow 0-\frac{\sqrt{3}}{2}=-\frac{1}{x^2}+1$

$\therefore x^2=\frac{2}{2+\sqrt{3}}=2(2-\sqrt{3})$ *

3. $n=40+45=85$

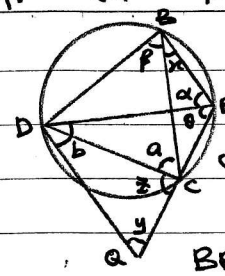
$\Sigma u=\Sigma x+\Sigma y=2380+45(52)=4720$

$\therefore \bar{u}=\frac{4720}{85}=55.53$ *

$\Sigma u^2=149370+45[52^2+8^2]=273930$

$\therefore s_u=\left[\frac{273930}{85}-\left(\frac{4720}{85}\right)^2\right]^{1/2}=11.80$ *

4. (a) $\beta=a=60^\circ$ (ΔBCD is equilateral)



$\angle BPD=a=60^\circ$ [$\angle$ in the same segment] *

$\angle CPD=\beta=60^\circ$ [$\angle$ in the same segment] *

(b) $\beta+x=z$ [ext. $\angle$ of cyclic BDCP]

$BD=CD$ [ΔBCD is equilateral]

$BP=CQ$ (given)

$\therefore \Delta PBD\equiv\Delta QCD$ [by SAS] *

(c) $\therefore DP=DQ \Rightarrow y=\theta=60^\circ$ [isosceles Δ]

$\therefore b=180^\circ-\theta-y=60^\circ$ [$\Sigma\angle$ in Δ]

$\therefore b=y=\theta=60^\circ \Rightarrow \Delta PDQ$ is equilateral *

5. X = no. of visitors purchase books.

$X\sim B(3,0.35)$:

$P(X\geq 2)=3(0.35)^2(0.65)+0.35^3=0.28175$ *

$Y\sim B(20000,0.35) \rightarrow Y\sim N(7000,4550)$

$P(Y\leq 7050) \rightarrow P(Y\leq 7050.5)$

$=P\left(Z\leq\frac{7050.5-7000}{\sqrt{4550}}\right)=0.77297$ *

6. (a) $P(M\cup N)=\frac{2}{3}=P(M)+P(N)$

$\therefore \frac{2}{3}=2.4x+2x \Rightarrow x=\frac{5}{33}$ *

(b) $P(M\cup N)=\frac{2}{3}=P(M)=2.4x$

$\therefore x=\frac{5}{18}$ *

(c) $P(M\cup N)=\frac{2}{3}=2.4x+2x-2.4x(2x)$

$\therefore 7.2x^2-6.6x+1=0$

$x=\frac{6.6\pm\sqrt{6.6^2-4(7.2)}}{2(7.2)}=0.7251$ or 0.1915 *

7. $W\sim N(300,50^2)$, $R\sim N(200,40^2)$

(a) $P(W<250)=P\left(Z<\frac{250-300}{50}\right)=0.1587$ *

$P(W<200\mid W<250)=\frac{P(Z<-2)}{0.1587}$

$=\frac{0.0228}{0.1587}=0.1437$ *

(b) $E(Y)=300-2(200)=-100$ *

$Var(Y)=50^2+(2)^2(40)^2=8900$ *

$Y\sim N(-100,8900)$

$P(W>2R)=P(Y>0)$

$=P\left(Z>\frac{100}{\sqrt{8900}}\right)=0.1446$ *

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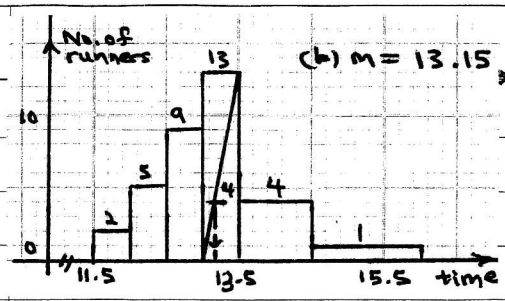
8. (a)



(b) $m = 13.15$ ✱

(c) max time $= x_4 = 12 + \frac{2}{5}(0.5) = 12.25$ ✱

9. (a) $2[s^6+c^6]-3[s^4+c^4]+1$

$= 2[s^2+c^2][s^4-s^2c^2+c^4]-3s^4-3c^4+1$

$= 2(1)(s^4-s^2c^2+c^4)-3s^4-3c^4+1$

$= 1-(s^4+2s^2c^2+c^4) = 1-(s^2+c^2)^2$

$= 1-(1)^2 = 0$ ✱

(b) $f(\theta) = 3\cos\theta - 4\sin\theta = R\cos(\theta+\alpha)$

$= R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$

$\therefore \left.\begin{array}{l} R\cos\alpha = 3 \\ R\sin\alpha = 4 \end{array}\right\} \tan\alpha = \frac{4}{3}$

$\therefore R\left(\frac{3}{5}\right) = 3 \Rightarrow R = 5$

$\therefore f(\theta) = 5\cos\left(\theta + \tan^{-1}\left(\frac{4}{3}\right)\right)$ ✱

$f(\theta)_{max} = 5(1) = 5$ ✱

$f(\theta)_{min} = 5(-1) = -5$ ✱

10. (a) $\frac{1}{2}(3)(3a+6a) = 1 \Rightarrow a = \frac{2}{27}$ ✱

(b) $P(X>b) = \frac{11}{27}$: $\frac{1}{2}(5-b)\left(\frac{2}{27}\right)(6+b+1) = \frac{11}{27}$

$\therefore b^2+2b-24 = 0 \Rightarrow (b+6)(b-4) = 0$

$[\because b>2]$: $b = 4$ ✱

(c) $P(1<X<4) = \frac{1}{2}(2)\left(\frac{2}{27}\right)(3+5) = \frac{16}{27}$ ✱

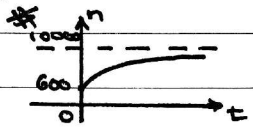
$P(X \le 3) = \frac{1}{2}\left(\frac{2}{27}\right)(3+4) = \frac{7}{27}$ ✱

11. $\frac{dn}{dt} \propto (N-n) \Rightarrow \frac{dn}{dt} = k(N-n)$ ✱

$\int \frac{dn}{N-n} = \int k\,dt \Rightarrow -\ln(N-n) = kt + c$

$\therefore N-n = e^{-kt}\cdot e^{-c}$ $[e^{-c} = A]$

$\therefore n = N - Ae^{-kt}$ ✱



$t=0$: $600 = 10000 - A$

$\therefore A = 9400$ ✱

$k=1$, For $n = \frac{N}{2}$: $t = -\ln(5000) + \ln 9400$

$= \ln(1.88)$ weeks

$\simeq 4.419$ days ✱

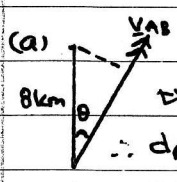
12. $\underline{V}_{AB} = \underline{V}_A - \underline{V}_B = 4\sqrt{2}\begin{pmatrix}\sin 45^\circ \\ \cos 45^\circ\end{pmatrix} - 5\begin{pmatrix}\sin 36^\circ \\ -\cos 36^\circ\end{pmatrix}$

$= \begin{pmatrix}1.0611 \\ 8.0451\end{pmatrix}$

$= \sqrt{1.0611^2 + 8.0451^2}$ kmh^{-1}, to N$\tan^{-1}\left(\frac{1.0611}{8.0451}\right)$E

$= 8.115$ kmh^{-1}, to N 7.5° E ✱

(a)



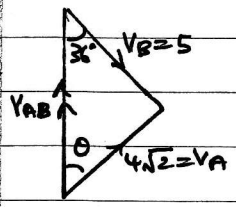
$\sin 7.5^\circ = 10.44$ km ✱

time taken $= \frac{80\cos 7.5^\circ}{8.115}$ h $= 9.774$h

$\therefore d_A = 4\sqrt{2}(9.774) = 55.29$ km ✱

$d_B = 5(9.774) = 48.87$ km ✱

(b) To intercept: V_{AB} (to N)



$\frac{\sin\theta}{5} = \frac{\sin 36^\circ}{4\sqrt{2}}$

$\therefore \theta = 31.3^\circ$

$\therefore$ Direction of A = N 31.3° E ✱

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