LEE HOCK LEONG
M.Sc. (Pure Maths) – CGPA 4
B.Sc with Ed. (1st Class Hons.)

SMJK JIT SIN (2006)
MATHEMATICS T (PAPER 1)

1. By using the laws of set algebra, prove that, for any sets A and B,
$(A \cup B) \cap (A \cup B') = A$. [3]

2. (a) Given that $\frac{1+3+5+\ldots+(2n-1)}{2+4+6+\ldots+2n} = \frac{115}{116}$. Find the value of n. [4]

(b) Find the volume of the solid of revolution generated when the region bounded by the curve $y = \frac{1}{1-x}$, the line $y = -2$, $y = -1$ and the y–axis is rotated 360° about the y–axis. [4]

3. The polynomial $f(x)$ leaves a remainder of -6 when divided by x, and a remainder 6 when divided by $(x-3)$. If the value of $f(x)$ is zero when $x = 1$ and $x = 2$, find the polynomial $f(x)$, given that it is of degree 3. [5]

4. (a) Find the value of x that satisfies the equation $\log_2 x = 2\log_8 (x+4)$. [6]

(b) Solve the inequality $3^x - 3(3^{-x}) + 2 < 0$. [6]

5. Find, in the form $x + yi$, the two complex numbers, z, satisfying the following simultaneous equations, $5z = \bar{z}(3+4i)$ and $z\bar{z} = 5$. [6]

6. Given that matrices $\boldsymbol{A} = \begin{pmatrix} 3 & -1 & 2 \\ -2 & 4 & 1 \\ 1 & -2 & -1 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 2 & 5 & 9 \\ 1 & 5 & 7 \\ 0 & -5 & -10 \end{pmatrix}$. Find the product $\boldsymbol{AB}$.

Hence, determine the values of x, y and z such that $\boldsymbol{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 25 \\ -14 \\ 5 \end{pmatrix}$. [8]

7. Given that the line $x - 2y = 5$ intersects another line $y = 2x - 1$ at the point H.

(a) Determine the coordinates of H. [3]

(b) A circle touches the line $y = 2x - 1$ at the point H and its centre lies on the line $x + y + 2 = 0$. Find the equation of the circle. [7]

(c) Show that the point $F(6, 2)$ lies outside the circle described in (b).
Hence, find the length of the tangent to the circle in (b) from the point F. [5]

8. The function f is defined by $f(x) = \begin{cases} \frac{x-|x|}{2} & , x < 0 \\ e^{1-x} + 1 & , x \geq 0 \end{cases}$.

Determine whether $f(x)$ is continuous at $x = 0$. Sketch the graph of $y = f(x)$. [6]

9. A water in the shape of an inverted circular cone has a radius of 20 m and a height of 12 m. Water is pumped into the tank at a rate of 40 m^3 min^{-1}. Calculate the rate of change of the depth of water in the tank when the depth of water is 4 m. [7]

10. The x–coordinate of the point of intersection of the curves $y = x^2 + \frac{1}{x}$ and $y = \frac{1}{x^2}$, where $x > 0$, is p. Show that $0.5 < p < 1$.

Use the Newton-Raphson method to determine the value of p correct to 3 decimal places and hence, find the point of intersection. [8]

11. Express $\frac{6x+4}{(1-2x)(1+3x^2)}$ in a partial fractions. [4]

Find $\int_1^2 \frac{6x+4}{(1-2x)(1+3x^2)}\,dx$.

Find also the coefficient of x^5 in the expansion of $\frac{6x+4}{(1-2x)(1+3x^2)}$, in ascending powers of x. [3]

12. For the function $f(x) = \frac{x^2}{x^2 - 5x + 6}$, find

(a) all the asymptotes,

(b) the stationary points, and the range of f.

Hence, sketch the curve. [12]

SMJK JIT SIN (2006)
MATHEMATICS T (PAPER 2)

1. Solve the equation $\cos 4\theta - \cos 2\theta = \sin 3\theta$ for $0^\circ \leq \theta \leq 180^\circ$. [4]

2. Prove that in any triangle ABC, $\tan A + \tan B + \tan C = \tan A \tan B \tan C$. [5]

Write down the square of the expression $\tan A + \tan B + \tan C$. [1]

Using the above relationships show that in any triangle ABC,

$\tan A + \tan B + \tan C - 2(\cot A + \cot B + \cot C)$

$= \tan A \cot B \cot C + \cot A \tan B \cot C + \cot A \cot B \tan C$. [5]

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3. Solve the differential equation $\frac{dy}{dx} = e^{2x+y}$ given that $y = 0$ when $x = 0$.

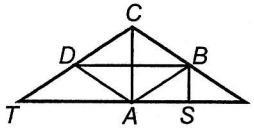
   Give your answer by expressing y in terms of x. [5]

4. At noon, two cars P and Q are at distances 300 m and 400 m from the point of intersection O of two straight roads crossing at a right angle and are approaching O at uniform speeds of 20ms^{-1} and 40ms^{-1} respectively. P travels due east and Q travels due north. Find the magnitude and direction of the velocity of P relative to Q.

   Hence, determine the shortest distance between the two cars. [7]

5. $ABCD$ is a rhombus and CD is produced to T so that $CD = DT$. Prove that the triangles BCD and ADT are congruent and that the diagonal DB is parallel to TA.

   The line through B parallel to AC meets TA produced at S. Prove that $\angle ASB = 90^\circ$.

   Write down the numerical value of $\frac{SA}{AT}$. [7]

6. The rate at which a body loses speed at any given instant as it travels through a resistive medium is given by $kv\ \text{ms}^{-1}$ where v is the speed of the body at that instant and k is a positive constant. The initial speed of the body is $a\ \text{ms}^{-1}$.

   (a) Find v in terms of t. [6]

   (b) Sketch the graph of v against t. [1]

   (c) Find the time taken for the body after the speed has decreased by 75%. [2]

7. A golfer observes that, when playing a particular hole at his local course, he hits a straight drive on 80% of the occasions when the weather is not windy but only 30% of the occasions when the weather is windy. Local records suggest that the weather is windy at 55% of all days.

   (a) Find the probability that on a randomly chosen day, the golfer will hit a straight drive at the hole. [2]

   (b) Given that he fails to hit a straight drive at the hole, calculate the probability that the weather is windy. [2]

8. X is a continuous random variable with probability density function defined by :

$$f(x) = \begin{cases} k(x+2)^2 & , -2 < x < 0 \\ 4k & , 0 \le x \le \frac{4}{3} \\ 0 & , \text{otherwise} \end{cases}$$

   (a) Find the value of the constant k. [4]

   (b) Sketch the graph of $y = f(x)$. [2]

   (c) Find the cumulative distribution function $F(x)$, and sketch its graph. [4]

   (d) Find $P(X < 0)$. Hence determine the median of X. Give your answer to three decimal places. [4]

24

9. A small store sells a certain size of nail either in a small packet at 50 cent per packet, or loose at RM 3.00 per kg. On any one shopping day the number, X, of packets sold in a random variable where $X \sim B(8, 0.6)$, and the weight, Y kg, of nails sold loose is a continuous random variable uniformly distributed in the interval $(1, 6)$. Find the probability that, on any one shopping day, the number of packets sold will be

   (a) more that one, (b) seven or fewer. [2]

   Find the probability that on any shopping day,

   (c) the weight of nails sold loose will be between 4 kg and 5 kg. [3]

   (d) the shop will sell exactly 2 packets of nails and less than 5 kg of nails sold loose. [2]

   Calculate the expected money received on any one shopping day from the sale of this size of nail in his store. [6]

10. In a factory, a certain machine produces empty cans whose masses are normally distributed with a mean of 32 g. Given that the probability of a can with mass more that 40.5 g is 0.045, determine the probability that its mass is less than 28 g. [5]

    A second machine fills a certain type of food into the cans. It is known that the mass of a can together with its content has a normal distribution with a mean of 550 g and a standard deviation of 13 g. By assuming the probability that the mass of a can and its content are independent, determine the probability that the mass of the content of a full can chose at random is less that 520 g. [3]

11. On a long train journey, a statistician is invited by a gambler to play a dice game. The game uses two unbiased ordinary dice which the statistician is to throw. If the total score is 12, the statistician is paid RM 3 by the gambler. However, if both or either dice show a '1', the statistician pays the gambler RM 2.

    Let X = Amount paid to the statistician by the gambler after the dice are thrown once. Determine the probability that $X = 6$, $X = 3$, and $X = -2$. [3]

    Find the expected value of X.

    If the statistician played the game 72 times, determine his expected loss.

    Find the amount that the RM 6 would have to be changed to in order to make the game unbiased. [6]

12. The following data indicate the number of cars passing through a small bridge in a small town of a small country in Europe over a period of 20 days.

59	20	48	73	23	56	31	36	34	61
36	44	33	25	65	38	21	40	33	24

    (a) Construct a stem plot to represent the above data. [2]

    (b) Comment on the distribution of the data. [1]

    (c) Determine the lower quartile, median and the upper quartile. Hence, find the interquartile range of the distribution. [2]

    (d) Draw a box plot to represent the data. [2]

1. $(A\cup B)\cap(A\cup B') = A\cup(B\cap B')$

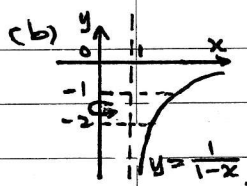
$= A\cup\phi = A$ ✱

2(a) $\dfrac{1+3+5+\dots+(2n-1)}{2+4+6+\dots+2n} = \dfrac{115}{116}$

$\therefore \dfrac{\frac{n}{2}[1+2n-1]}{\frac{n}{2}[2+2n]} = \dfrac{115}{116}$

$116n = 115(n+1) \Rightarrow n = 115$ ✱

(b)



$V_y = \pi\int_{-2}^{-1}\left[1-\frac{1}{y}\right]^2 dy$

$= \pi\int_{-2}^{-1}\left[1-\frac{2}{y}+\frac{1}{y^2}\right]dy$

$= \pi\left[y-2\ln|y|-\frac{1}{y}\right]_{-2}^{-1}$

$= \pi\left[(-1+2)-2\ln\left(\frac{1}{2}\right)-\left(-1+\frac{1}{2}\right)\right]$

$\therefore V_y = \pi\left[\frac{3}{2}+2\ln 2\right]$ ✱

3. $f(1)=0, f(2)=0$: $f(x)=a(x-1)(x-2)(x-k)$

$f(0)=-6$: $-6=-2ak \Rightarrow ak = 3$

$f(3)=6$: $6=a(2)(1)(3-k)=6a-2ak$

$\therefore 6=6a-6 \Rightarrow a=2$, $\therefore k=\frac{3}{2}$

$\therefore f(x)=(x-1)(x-2)(2x-3)$ ✱

4.(a) $\log_2 x = 2\log_8(x+4) = 2\left[\frac{\log_2(x+4)}{3}\right]$

$\therefore \log_2 x^3 = \log_2(x+4)^2$

$\therefore x^3=(x+4)^2 \Rightarrow x^3-x^2-8x-16=0$

$\therefore (x-4)(x^2+3x+4)=0$

$\therefore x=4$ ✱ $[\because x\in\mathbb{R},\ x>0]$

(b) $3^x-3(3)^{-x}+2<0$, let $u=3^x$

$\therefore u-\frac{3}{u}+2<0 \Rightarrow u^2+2u-3<0$

$\therefore (u+3)(u-1)<0 \Rightarrow (3^x+3)(3^x-1)<0$

[since $3^x>0, \forall x\in\mathbb{R} \Rightarrow 3^x+3>0$]

$\therefore 3^x-1<0 \Rightarrow 3^x<1$

$\{x : x<0\}$ ✱

5. Let $z=x+yi$: $(x,y\in\mathbb{R})$

$5z=z^*(3+4i)$: $5(x+yi)=(x-yi)(3+4i)$

Re : $5x=3x+4y$ Im : $5y=-3y+4x$

$\therefore x=2y$ $\therefore x=2y$

$zz^*=5$: $x^2+y^2=5 \Rightarrow 4y^2+y^2=5$

$\therefore y^2=1 \Rightarrow y=1$ or -1

$\therefore z=2+i$ or $-2-2i$ ✱

6. $AB = \begin{bmatrix} 6-1+0 & 15-5-10 & 27-7-20 \\ -4+4+0 & -10+20-5 & -18+28-10 \\ 2-2+0 & 5-10+5 & 9-14+10 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ ✱

$AB=5I \Rightarrow A^{-1}=\frac{1}{5}B$

$A\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 25 \\ -14 \\ 5 \end{pmatrix}$: $\begin{pmatrix} x \\ y \\ z \end{pmatrix}=A^{-1}\begin{pmatrix} 25 \\ -14 \\ 5 \end{pmatrix}$

$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix}=\frac{1}{5}\begin{pmatrix} 7 & 5 & 9 \\ 1 & 5 & 7 \\ 0 & -5 & -10 \end{pmatrix}\begin{pmatrix} 25 \\ -14 \\ 5 \end{pmatrix}=\begin{pmatrix} 5 \\ -2 \\ 4 \end{pmatrix}$

$\therefore x=5,\ y=-2,\ z=4$ ✱

7.(a) $x-2y=5$: $x-2(2x-1)=5 \Rightarrow x=-1$

$\therefore H(-1,-3)$ ✱

(b) Let $C(c,-c-2)$: $m_{CH}=-\frac{1}{2}=\frac{-c-2+3}{c+1}$

$\therefore c+1=2c-2 \Rightarrow c=3$ $\therefore C(3,-5)$

$\therefore$ Circle : $(x-3)^2+(y+5)^2=(3+1)^2+(-5+3)^2$

$\therefore x^2+y^2-6x+10y+14=0$ ✱

(c) $FC=\sqrt{3^2+7^2}=\sqrt{58}$ $(>\sqrt{20}=r)$

$\therefore$ F lies outside the circle ✱

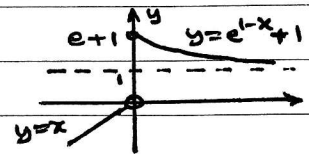
tangent length $=\sqrt{58-20}=\sqrt{38}$ ✱

8. $f(x)=\begin{cases} \frac{x-|x|}{2}, & x<0 \\ e^{1-x}+1, & x\geq 0 \end{cases} = \begin{cases} x, & x<0 \\ e^{1-x}+1, & x\geq 0 \end{cases}$

$f(0)=e+1$, $\lim_{x\to 0^-} f(x)=0$

$\therefore \lim_{x\to 0^-} f(x)\neq f(0)$

$\therefore f$ is not continuous at $x=0$ ✱



9.

$\frac{h}{r}=\frac{12}{20} \Rightarrow r=\frac{5}{3}h$

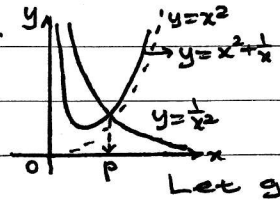
$V=\frac{\pi}{3}r^2h \Rightarrow V=\frac{\pi}{3}\left(\frac{5}{3}h\right)^2h$

$\therefore \frac{dV}{dh}=\frac{25\pi}{9}h^2$

$\frac{dV}{dt}=\frac{dV}{dh}\times\frac{dh}{dt}$: $40=\frac{25\pi}{9}(4)^2\times\frac{dh}{dt}$

$\therefore \frac{dh}{dt}=\frac{9}{10\pi}$ m per min ✱

10.



Let $f(x)=x^2+\frac{1}{x}-\frac{1}{x^2}$

$f(0.5).f(1)=\left(-\frac{7}{4}\right)(1)<0$

$\therefore 0.5<p<1$ ✱

Let $g(x)=x^2.f(x)=x^4+x-1$

By NRM : $x'=x-\frac{g(x)}{g'(x)}=x-\frac{x^4+x-1}{4x^3+1}$

Let $x_0=0.72$: $x_1=0.72-\frac{(-0.01126)}{2.49299}=0.72452$

$\simeq 0.725$

$x_2=0.72452-\frac{7\times10^{-5}}{2.52129}=0.72449 \simeq 0.724$

$x_3=0.72449-\frac{(-5\times10^{-6})}{2.52110}=0.72449 \simeq 0.724$

$\therefore p=0.724$ (3dp) ✱

11. Let $f(x) = \frac{6x+4}{(1-2x)(1+3x^2)} \equiv \frac{A}{1-2x} + \frac{Bx+C}{1+3x^2}$

$\equiv \frac{A(1+3x^2)+(Bx+C)(1-2x)}{(1-2x)(1+3x^2)}$

$\therefore 6x+4 \equiv A(1+3x^2) + (Bx+C)(1-2x)$

$x=\frac{1}{2}$: $3+4 = A(1+\frac{3}{4}) \Rightarrow A=4$

$[x^2]$: $0 = 3A - 2B \Rightarrow B = 6$

$[x^0]$: $4 = A + C \Rightarrow C = 0$

$\therefore f(x) = \frac{4}{1-2x} + \frac{6x}{1+3x^2}$ ✱

(i) $\int_1^2 f(x)\,dx = \left[-2\ln(2x-1) + \ln(1+3x^2)\right]_1^2$

$= -2\ln(\frac{3}{1}) + \ln(\frac{13}{4}) = \ln(\frac{13}{36})$ ✱

(ii) $\quad$ of $x^5 = 4(2x)^5 + 6x(3x^2)^2 = 182x^5$

$\therefore$ coef. of $x^5 = 182$ ✱

12. $f(x) = \frac{x^2}{x^2-5x+6} = 1 + \frac{5x-6}{(x-2)(x-3)}$

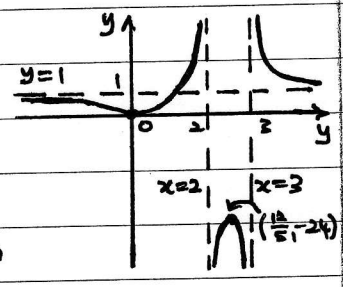
(i) asymptotes: $y=1$, $x=2$, $x=3$ ✱

(ii) $f'(x) = \frac{(x^2-5x+6)(5)-(5x-6)(2x-5)}{(x^2-5x+6)^2}$

for $f'(x)=0$: $-5x^2+12x=0 \Rightarrow x=0$ or $\frac{12}{5}$

$\therefore$ stationary points

$= (0,0), (\frac{12}{5}, -24)$ ✱



(iii) Let $y=f(x)$:

$yx^2-5yx+6y = x^2$

$\therefore (y-1)x^2 - 5yx + 6y = 0$

For all real values of x: $b^2-4ac \geq 0$

$25y^2 - 4(y-1)(6y) \geq 0$

$\therefore y(y+24) \geq 0$

$\therefore$ Range of f: $\{y : y \leq -24 \text{ or } 0 \leq y\}$ ✱

1. $\cos 4\theta - \cos 2\theta = \sin 3\theta$

$\therefore -2\sin 3\theta \sin\theta = \sin 3\theta$

$\therefore \sin 3\theta[2\sin\theta + 1] = 0$

$\therefore \sin 3\theta = 0$ or $\sin\theta = -\frac{1}{2}$

$\therefore 3\theta = 0°, 180°, 360°, 540°$ | $\therefore$ no solution ✱

$\therefore \theta = 0°, 60°, 120°, 180°$ ✱

2. $\tan(A+B+C) = \tan 180°$

$\therefore \frac{\tan A + \tan(B+C)}{1-\tan A\tan(B+C)} = 0$

$\therefore \tan A + \frac{\tan B + \tan C}{1-\tan B\tan C} = 0$

$\therefore \tan A - \tan A\tan B\tan C + \tan A + \tan B = 0$

$\therefore \tan A + \tan B + \tan C = \tan A\tan B\tan C$ ✱

$[\tan A + \tan B + \tan C]^2 = \tan^2 A + \tan^2 B + \tan^2 C$

$+ 2[\tan A\tan B + \tan A\tan C + \tan B\tan C]$ ✱

$\tan A + \tan B + \tan C - 2[\cot A + \cot B + \cot C]$

$= \tan A + \tan B + \tan C - 2\left[\frac{\tan B\tan C + \tan A\tan C + \tan A\tan B}{\tan A\tan B\tan C}\right]$

$= \frac{(\tan A + \tan B + \tan C)^2 + \tan^2 A + \tan^2 B + \tan^2 C - (\tan A + \tan B + \tan C)^2}{\tan A\tan B\tan C}$

$= \frac{\tan^2 A + \tan^2 B + \tan^2 C}{\tan A\tan B\tan C}$

$= \tan A\cot B\cot C + \tan B\cot A\cot C + \tan C\cot A\cot B$ ✱

3. $\frac{dy}{dx} = e^{2x+y} \Rightarrow \int e^{-y}\,dy = \int e^{2x}\,dx$

$\therefore -e^{-y} = \frac{1}{2}e^{2x} + c$

$x=0, y=0$: $-1 = \frac{1}{2} + c \Rightarrow c = -\frac{3}{2}$

$\therefore -e^{-y} = \frac{1}{2}(e^{2x} - 3) \Rightarrow e^y = \frac{2}{3-e^{2x}}$

$\therefore y = \ln\left[\frac{2}{3-e^{2x}}\right]$ ✱

4. $\underline{V}_{PQ} = \underline{V}_P - \underline{V}_Q = 20\begin{pmatrix}1\\0\end{pmatrix} - 40\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}20\\-40\end{pmatrix}$

$= 20\sqrt{1^2+(-2)^2}\ ms^{-1}$, to $S\tan^{-1}(\frac{1}{2})E$

$= 20\sqrt{5}\ ms^{-1}$, to $S\ 26.57°\ E$ ✱

$S_{PQ} = \begin{pmatrix}-300\\400\end{pmatrix} + t\begin{pmatrix}20\\-40\end{pmatrix}$

For SD: $(S_{PQ} \cdot V_{PQ} = 0)$

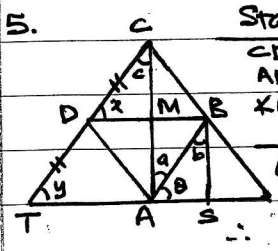
$-6000 - 16000 + t(20^2+40^2) = 0$

$\therefore t = 11$

$\therefore SD = \left|\begin{pmatrix}-300\\400\end{pmatrix} + 11\begin{pmatrix}20\\-40\end{pmatrix}\right| = \left|20\begin{pmatrix}3\\-2\end{pmatrix}\right|$

$= 20\sqrt{13}\ m$ ✱

5.



Statement	Reason
$CD = DT$	given
$AD = BC$	ABCD rhombus
$\angle ADT = \angle BCD$	corresponding angles (AD//BC)
$\Delta BCD \equiv \Delta ADT$	By SAS ✱

$\therefore x = y \Rightarrow$ C. Angles, DB//TA ✱

$\angle CMD = 90°$ (ABCD rhombus)

$\angle CMD = \angle CAT = 90°$ [C. Angles, DB//TA]

$\angle CAT = \angle BSA = 90°$ ✱ [C. Angles AC//SB]

$c = a$ (Alternate angles, DC//AB]

$a = b$ (Alternate angles, AC//SB] $\therefore c = b$

$y = \theta$ (Corresponding angles, TC//AB)

$\therefore \Delta TAC \sim \Delta ASB \Rightarrow \frac{SA}{AT} = \frac{AB}{TC} = \frac{1}{2}$ ✱

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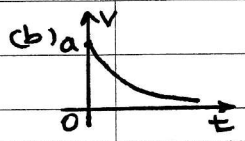
6. (a) $\frac{dv}{dt} = -kv \Rightarrow \int \frac{1}{v}dv = \int -k\,dt$

$\ln v = -kt + c$ — ①

$t=0, v=a$: $\ln a = c$ — ②

① − ②: $\ln\left(\frac{v}{a}\right) = -kt \Rightarrow v = ae^{-kt}$ ✱

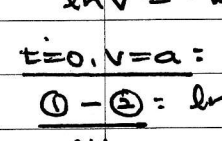
(b)



(c) For $v = \frac{1}{4}a$:

$t = -\frac{1}{k}\ln\left(\frac{1}{4}\right) = \frac{1}{k}\ln 4$ ✱

7.



(a) $P(S) = 0.55(0.3) + 0.45(0.8)$

$= 0.525$ ✱

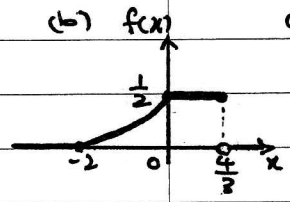
(b) $P(W|S) = \frac{0.55(0.3)}{0.525} = \frac{11}{35}$ ✱

8. (a) $\int_{-2}^{0} k(x+2)^2dx + \int_0^{4/3} 4k\,dx = 1$

$\frac{k}{3}\left[(x+2)^3\right]_{-2}^{0} + 4k\left[x\right]_0^{4/3} = 1$

$\frac{k}{3}[8-0] + 4k\left[\frac{4}{3}-0\right] = 1 \Rightarrow k = \frac{1}{8}$ ✱

(b)



(c) for $-2 < x < 0$:

$P(X \le x) = \int_{-2}^{x} \frac{1}{8}(x+2)^2dx$

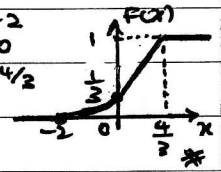
$= \frac{1}{24}\left[(x+2)^3\right]_{-2}^{x} = \frac{1}{24}(x+2)^3$

For $0 \le x \le \frac{4}{3}$:

$P(X \le x) = \int_{-2}^{0} \frac{1}{8}(x+2)^2dx + \int_0^x \frac{1}{2}dx$

$= \frac{1}{24}(8) + \frac{1}{2}\left[x\right]_0^x = \frac{1}{2}x + \frac{1}{3}$

$\therefore F(x) = \begin{cases} 0 & x \le -2 \\ \frac{1}{24}(x+2)^3 & -2 < x < 0 \\ \frac{1}{2}x + \frac{1}{3} & 0 \le x \le 4/3 \\ 1 & 4/3 < x \end{cases}$ ✱



(d) $P(X<0) = \frac{1}{3}$ ✱

$\therefore P(X<m) = \frac{1}{2} = \frac{1}{2} + \frac{1}{3} \Rightarrow m = \frac{1}{3}$

$\therefore m = 0.333$ (3dp) ✱

9. $X \sim B(8, 0.6)$ $Y \sim U(1, 6)$

(a) $P(X>1) = 1 - [0.4^8 + 8(0.4)^7(0.6)] = 0.991$ ✱

(b) $P(X \le 7) = 1 - (0.6)^8 = 0.983$ ✱

(c) $P(4 < Y < 5) = 1\left(\frac{1}{5}\right) = \frac{1}{5}$ ✱

(d) $P(X=2 \text{ and } Y<5)$

$= \left[{}^8C_2(0.6)^2(0.4)^6\right]\left[(5-1)\left(\frac{1}{5}\right)\right] = 0.033$ ✱

(e) $E[0.5X + 3Y] = 0.5(8)(0.6) + 3\left(\frac{1+6}{2}\right)$

$= \text{RM}12.90$ ✱

10. X = mass of a empty can, $X \sim N(32, \sigma^2)$

$P(X > 40.5) = 0.045 = P\left(Z > \frac{40.5-32}{\sigma}\right)$

$\therefore \frac{8.5}{\sigma} = 1.695 \Rightarrow \sigma = 5.015$

$P(X < 28) = P\left(Z < \frac{28-32}{5.015}\right) = 0.21255$ ✱

Y = mass of filled can, $Y \sim N(550, 13^2)$

$P(Y - X < 520)$ | $Y - X \sim N(550-32, 13^2 + 5.015^2)$

$= P\left(Z < \frac{520-518}{\sqrt{194.15}}\right)$ | $\therefore Y - X \sim N(518, 194.15)$

$= P(0.1435) = 0.55705$ ✱

11. (a) $P(X=6) = \left(\frac{1}{6}\right)^2 = \frac{1}{36}$ ✱

(b) $P(X=3) = \left(\frac{1}{6}\right)^2(5) = \frac{5}{36}$ ✱

(c) $P(X=-2) = \left(\frac{1}{6}\right)^2(11) = \frac{11}{36}$ ✱

$P(X=0) = \frac{19}{36}$

$\therefore E(X) = \frac{1}{36}\left[(-2)(11) + 0 + 3(5) + 6(1)\right] = -\frac{1}{36}$ ✱

Expected loss $= 72\left(\frac{1}{36}\right) = \text{RM}2$ ✱

Unbiased: $E(X) = 0$

$\frac{1}{36}\left[-22 + 0 + 15 + k(1)\right] = 0 \Rightarrow k = 7$

$\therefore$ Amount change to RM7 ✱

12(a)

2	0 1 3 4 5
3	1 3 3 4 6 6 8
4	0 4 8
5	6 9
6	1 5
7	3

key: 2|0 means 20 ✱

(b) Distribution is right-skewed ✱

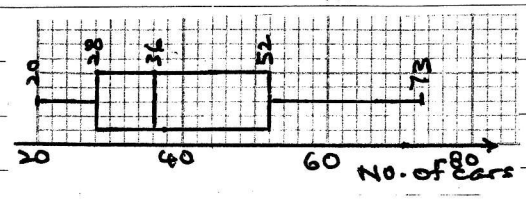
(c) $Q_1 = \frac{x_5 + x_6}{2} = \frac{1}{2}(25+31) = 28$ ✱

$m = \frac{x_{10} + x_{11}}{2} = \frac{1}{2}(36+36) = 36$ ✱

$Q_3 = \frac{x_{15} + x_{16}}{2} = \frac{1}{2}(48+56) = 52$ ✱

$IQR = 52 - 28 = 24$ ✱

(d)



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