## DAERAH MUAR, JOHOR (2006)
## MATHEMATICS T (PAPER 1)

1. Using the laws of the algebra of sets, show that $A' \cap (B - A)' = (A \cup B)'$. [4]

2. If $y = \sin[\ln(1 + x)]$, show that $(1 + x)^2 \frac{d^2y}{dx^2} + (1 + x)\frac{dy}{dx} + y = 0$. [4]

3. The equation of a curve is $2x^2 + 8xy + 5y^2 + 3 = 0$.
Find the equations of the two tangents which are parallel to the $x$–axis. [5]

4. Find the sum to $n$ terms of the series $3 - x + \frac{x^2}{3} - \frac{x^3}{3^2} + \ldots + \frac{(-1)^{r-1}x^{r-1}}{3^{r-2}} + \ldots$.

Given that the series is convergent, state the set of possible values of $x$, and the sum to infinity in terms of $x$. [6]

5. Solve the inequality $|5x - 7| > 2x + 1$. [5]

6. The coordinates of the vertices $O$, $A$ and $B$ of a triangle are $(0, 0)$, $(12, 0)$ and $(0, 16)$ respectively.

   (a) Verify that the point with coordinates $(4, 4)$ is equidistant from the three sides of the triangle $OAB$. [5]

   (b) Obtain the equation of the inscribed circle of the triangles $OAB$. [4]

7. (a) Matrices $P$ and $Q$ are given as $P = \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}$, $Q = \begin{pmatrix} x & y \\ 1 & -1 \end{pmatrix}$, where $x$ and $y$ are real numbers. Find the diagonal matrix $S$ such that $PSP^{-1} = Q$. [5]

   (b) If $M = \begin{pmatrix} 1 & 2 & 0 \\ -1 & 0 & 2 \\ 1 & 1 & -1 \end{pmatrix}$, show that $M^3 = M$. Hence, find $M^{20}$. [5]

8. Sketch the curves $y = x^2(5x - 6)$ and $y = x(4x - 5)$ on the same diagram. [3]

   Calculate the area of the region bounded by the curves. [5]

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9. The complex numbers $z_1$ and $z_2$ satisfy the equation $z^2 = -3 + 4i$.

   (a) Express $z_1$ and $z_2$ in the form $a + bi$, where $a$ and $b$ are real numbers. [6]

   (b) Represent $z_1$ and $z_2$ in an Argand diagram. [1]

   (c) For each of $z_1$ and $z_2$, find the modulus, and the argument in radians. [4]

10. (a) The function $f$ and $g$ are defined by $f(x) = e^x$ and $g(x) = 1 - x$, for $x \in \mathcal{R}$.

   (i) State the ranges of $f$ and $g$. [1]

   (ii) Find $f^{-1}(x)$ and hence obtain $gf^{-1}(x)$. [5]

   (b) If function $h$ is defined as $h(x) = \begin{cases} Ax + 2 & , -3 \le x \le 1 \\ x^2 + Bx + A & , 1 < x \le 2 \\ 1 - \frac{1}{x} & , 2 < x \le 5 \end{cases}$.

   Find the values of $A$ and $B$ such that $h$ is continuous in the interval $[-3, 5]$. [5]

11. Express $f(x) = \frac{3x^2 - 16x + 11}{(2 - x)^2(1 + 4x)}$ in partial fractions. [5]

   Hence, find the expansion of $f(x)$ in the ascending powers of $x$ up to the term in $x^3$. [6]

   State the set of the values of $x$ for which the expansion is valid. [2]

12. Sketch the graph of $y = e^x$ and $y = 7 - 3x$ on the same coordinate axes. [3]

   Show that the equation $e^x = 7 - 3x$ has a root that lies between 1 and 1.5. [4]

   By using the Newton-Raphson method, determine the root of the equation correct to three decimal places. [6]

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## MATHEMATICS T (PAPER 2)

1. $PA$ and $PB$ are tangents to a circle from an external point $P$. $C$ is a point on the minor arc of $AB$. $AC$ meets $BP$ at $D$ and $BC$ meets $AP$ at $E$.
Show that $\angle PAC + \angle PBC + \angle DCE = 180^\circ$. [5]

2. In a triangle $ABC$, $D$ is the midpoint of $BC$ and $E$ is the midpoint of $CA$.
The lines $AD$ and $BE$ intersect at $X$. Prove that

   (a) the area of the triangle $DEC$ is equal to half the area of the triangle $ABD$, [4]

   (b) the triangles $AXE$ and $BXD$ are equal in area. [3]

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3. $A$, $B$, $C$, and $D$ are points with position vectors $\underline{i} + 2\underline{j}$, $3\underline{i}$, $4\underline{i} + 5\underline{j}$ and $7\underline{i} + 4\underline{j}$ respectively.
   (a) Show that $AB$ is perpendicular to $AC$. [3]
   (b) Show that $AC$ and $BD$ are parallel. [2]
   (c) Find the angle between $BC$ and $BD$. [3]

4. Ship $A$ which is travelling at a speed of $15\text{ km h}^{-1}$ in the direction $S45^\circ E$ picks up ship $B$ on its radar screen. At that instant, ship $B$ which is travelling at a speed of $20\text{ km h}^{-1}$ in the direction $S75^\circ E$ is 50 km in the direction $S40^\circ W$ of ship $A$. If both ships maintain these velocities, find the velocity of $A$ relative to $B$ and the shortest distance between the ships in the ensuing motion. [8]

5. Prove that $\dfrac{1}{\sin 2x} + \dfrac{1}{\tan 2x} = \cot x$. [3]

   Hence,
   (a) show that $\cot 15^\circ = 2 + \sqrt{3}$. [3]
   (b) solve the equation $\dfrac{1}{\sin 3\theta} + \dfrac{1}{\tan 3\theta} = 2$, for $0 < \theta < 180^\circ$. [3]

6. A territory will support a maximum population of $P_0$. Let the ratio of the population $P$ to the maximum population $P_0$ be $p$.
   The rate of change of this ratio $p$ is proportional to the product of $p$ and the difference between $p$ and 1. Write down a differential equation in $p$. [2]

   Show that the growth of population is greatest when $P$ is $\frac{1}{2}P_0$. [3]

   The initial population is $\frac{1}{4}P_0$, and reaches $\frac{1}{2}P_0$ after 20 years.

   Find the time for the population to reach $\frac{7}{8}P_0$? [7]

   Find the value of the maximum growth of the population, correct to 3 decimal places. [2]

7. Fifteen athletes compete in the triple-jump event at a sport meeting. The distances ($D$ m) that they jump are normally distributed following $D \sim N(17.1, 0.04)$.
   In the first half of the competition, the athletes have to jump a distance of 17.15 m to qualify for the final section.
   How many athletes would you expect to achieve this, to the nearest integer? [5]

8. Three men, $A$, $B$ and $C$ agree to meet at the theatre. The man $A$ cannot remember whether they agreed to meet at the Cathay or the Rex and tosses a coin to decide which theatre to go to. The man $B$ also tosses a coin to decide between the Rex and the Victory. The man $C$ tosses a coin to decide whether to go to the Cathay or not and in this latter case he tosses again to decide between the Rex and the Victory.

   Find the probability that
   (a) $A$, $B$ and $C$ all meet, [2]
   (b) $A$, $B$ and $C$ all go to different places, [2]
   (c) at least two meet. [2]

9. The number of people entering a railway station between the hours of 7.30 and 9.30 am on a weekday follows a Poisson distribution with a mean two people per minute. Find the probability that
   (a) five people enter in one minute, [2]
   (b) less than four people enter in a two-minute period. [3]

10. The continuous random variable $X$ has probability density function given by

$$f(x) = \begin{cases} k(2-x) & ,0 \le x \le 2 \\ 0 & ,\text{otherwise} \end{cases}, \text{ where k is a constant.}$$

   (a) Find the value of $k$. [2]
   (b) Find the cumulative distribution function of $X$. [4]

   The continuous random variable $Y$ is given by $Y = 1 - \frac{X}{2}$.

   Show that $P(Y < y) = y^2$, where $0 \le y \le 1$. [4]

11. A discrete random variable $X$ has the probability distribution given in the following table.

| $x$ | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| $P(X = x)$ | $p$ | 0.2 | 0.3 | $q$ |

   (a) Given that $E(X) = 4$, find $p$ and $q$. [6]
   (b) Show that $Var(X) = 1$. [3]
   (c) Find $E(|X - 4|)$. [2]

12. The following data show the masses (kg), of fish caught by 25 fishermen on a particular day.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 8 | 7 | 4 | 4 | 18 | 9 | 14 | 17 | 11 | 5 | 4 | 5 |
| 7 | 10 | 12 | 14 | 10 | 9 | 29 | 7 | 6 | 5 | 20 | 15 | |

   (a) Find the mean and standard deviation. [5]
   (b) Find the percentage of the fishermen with the mass of fish within one standard deviation from the mean. [3]
   (c) Construct a frequency distribution by grouping the masses of fish according to class intervals 1 – 5, 6 – 10, 11 – 15, 16 – 20, and 21 – 30.

   Hence, plot a histogram corresponding to this grouped data. [4]

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1. $A' \cap (B-A)' = [A \cup (B \cap A')]'$

$= [(A \cup B) \cap (A \cup A')]'$

$= [(A \cup B) \cap \varepsilon]' = (A \cup B)'$ ✱

2. $y = \sin[\ln(1+x)] \Rightarrow y' = \cos[\ln(1+x)] \cdot \frac{1}{1+x}$

$\therefore (1+x)^2 (y')^2 = 1 - \sin^2[\ln(1+x)] = 1 - y^2$

$\therefore (1+x)^2 \cdot 2y'y'' + 2(1+x)(y')^2 = -2yy'$

$\therefore (1+x)^2 y'' + (1+x)y' + y = 0$ ✱

3. $2x^2 + 8xy + 5y^2 + 3 = 0$ — ①

$4x + 8[xy' + y] + 10y' = 0$ — ②

tangent // x-axis: $(y' = 0)$

②: $4x + 8y = 0 \Rightarrow x = -2y$

$\therefore$ ①: $8y^2 - 16y^2 + 5y^2 + 3 = 0$

$\therefore y^2 = 1 \Rightarrow y = \pm 1$ ✱ [tangents]

4. $S_n = \frac{3[1-(\frac{x}{3})^n]}{1-\frac{x}{3}} = \frac{9[1-(\frac{x}{3})^n]}{3-x}$ ✱

Series converges: $|\frac{x}{3}| < 1 \Rightarrow |x| < 3$

$\therefore \{x : -3 < x < 3\}$ ✱

$\therefore S_\infty = \frac{3}{1-\frac{x}{3}} = \frac{9}{3-x}$ ✱

5. $|5x-7| > 2x+1$

$\therefore 5x-7 < -(2x+1)$ or $2x+1 < 5x-7$

$\therefore x < \frac{5}{7}$ or $\frac{8}{3} < x$

$\therefore \{x : x < \frac{5}{7} \text{ or } \frac{8}{3} < x\}$ ✱

6.(a) AB: $\frac{x}{12} + \frac{y}{16} = 1 \Rightarrow 4x + 3y = 48$

$d(4,4) \text{ to } AB = \frac{|4(4)+3(4)-48|}{\sqrt{16+9}} = 4$

$\because d(4,4) \text{ to } OA = d(4,4) \text{ to } OB = 4$

$\therefore (4,4)$ is equidistance from the 3 sides of $\Delta OAB$. ✱

(b) Circle: $(x-4)^2 + (y-4)^2 = 4^2$

$\therefore x^2 + y^2 - 8x - 8y + 16 = 0$ ✱

7.(a) $PSP^{-1} = Q$: $PS = QP$, [Let $S = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$]

$\begin{pmatrix} 2a & 3b \\ -a & b \end{pmatrix} = \begin{pmatrix} 2x-y & 3x+y \\ 2 & 2 \end{pmatrix}$

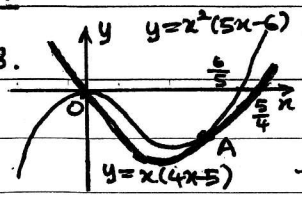
$\therefore a = -3$ and $b = 2 \Rightarrow S = \begin{pmatrix} -3 & 0 \\ 0 & 2 \end{pmatrix}$ ✱

(b) $M^2 = \begin{pmatrix} -1 & 2 & 4 \\ 1 & 0 & -2 \\ -1 & 1 & 3 \end{pmatrix}$, $M^3 = \begin{pmatrix} 1 & 2 & 0 \\ -1 & 0 & 3 \\ 1 & 1 & -1 \end{pmatrix} = M$ ✱

$M^3 = (M^3)^6 M^2 = M^6 M^2 = (M^3)^2 M^2$

$= M^2 \cdot M^2 = M^3 M = M^2 = \begin{pmatrix} -1 & 2 & 4 \\ 1 & 0 & -2 \\ -1 & 1 & 3 \end{pmatrix}$ ✱

8.




For A: $x^2(5x-6) = x(4x-5)$

$x(x-1)^2 = 0$

$\therefore x_A = 1 \Rightarrow A(1,-1)$

$\text{Area} = \int_0^1 [x^2(5x-6) - x(4x-5)]\,dx$

$= \int_0^1 5(x^3 - 2x^2 + x)\,dx = 5\left[\frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2}\right]_0^1$

$= 5\left(\frac{1}{4} - \frac{2}{3} + \frac{1}{2}\right) - 0 = \frac{5}{12}$ ✱

9.(a) Let $z = a + bi$: $a^2 - b^2 + 2abi = -3 + 4i$

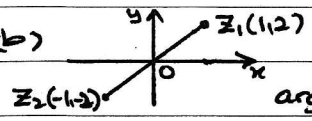
$\therefore a^2 - b^2 = -3$ and $b = \frac{2}{a}$

$\therefore a^2 - \frac{4}{a^2} = -3 \Rightarrow a^4 + 3a^2 - 4 = 0$

$\therefore (a^2+4)(a^2-1) = 0 \Rightarrow a = \pm 1$, [$\because a \in \mathbb{R}$]

$\therefore z_1 = 1 + 2i$, $z_2 = -1 - 2i$ ✱

(b)




(c) $|z_1| = |z_2| = \sqrt{5}$ ✱

$\arg(z_1) = \tan^{-1}(2) = 1.107$ ✱

$\therefore \arg(z_2) = \tan^{-1}(2) = -2.034$ ✱

10.(a) (i) $R_f: \{x : x > 0\}$, $R_g: x \in \mathbb{R}$ ✱

(ii) $f^{-1}: x \to \frac{1}{2}\ln x$, $x > 0$ ✱

$gf^{-1}(x) = g(\frac{1}{2}\ln x) = 1 - \frac{1}{2}\ln x$, $x > 0$ ✱

(b) $h(1) = \lim_{x \to 1^+} h(x)$: $A + 2 = 1 + B + A \Rightarrow B = 1$ ✱

$h(2) = \lim_{x \to 2^+} h(x)$: $4 + (1)(2) + A = 1 - \frac{1}{2} \Rightarrow A = -\frac{11}{2}$ ✱

11. $f(x) = \frac{3x^2 - 16x + 11}{(2-x)^2(1+4x)} \equiv \frac{A}{2-x} + \frac{B}{(2-x)^2} + \frac{C}{1+4x}$

$\equiv \frac{A(2-x)(1+4x) + B(1+4x) + C(2-x)^2}{(2-x)^2(1+4x)}$

$\therefore 3x^2 - 16x + 11 \equiv A(2-x)(1+4x) + B(1+4x) + C(2-x)^2$

$x = 2$: $12 - 32 + 11 = B(9) \Rightarrow B = -1$

$x = -\frac{1}{4}$: $\frac{3}{16} + 4 + 11 = C(\frac{9}{4})^2 \Rightarrow C = 3$

$[x^2]$: $3 = -4A + C \Rightarrow A = 0$

$\therefore f(x) = \frac{3}{1+4x} - \frac{1}{(2-x)^2}$ ✱

$f(x) = 3(1+4x)^{-1} - \frac{1}{4}(1 - \frac{x}{2})^{-2}$

$= 3[1 - 4x + (4x)^2 - (4x)^3 + \dots]$

$- \frac{1}{4}[1 + 2(\frac{x}{2}) + 3(\frac{x}{2})^2 + 4(\frac{x}{2})^3 + \dots]$

$= \frac{11}{4} - \frac{49}{4}x + \frac{765}{16}x^2 - \frac{1537}{8}x^3 + \dots$ ✱

For Exp. Valid: $|4x| < 1$ and $|\frac{x}{2}| < 1$

$\therefore |x| < \frac{1}{4}$ and $|x| < 2$

$\therefore \{x : -\frac{1}{4} < x < \frac{1}{4}\}$ ✱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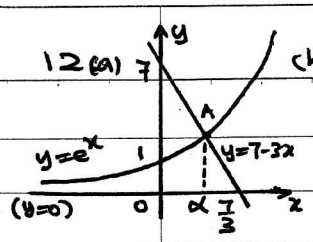
12 (a)




(b) Let $f(x)=e^x+3x-7$

$f(1).f(1.5)$

$=(-1.28)(1.98)<0$

$\therefore\ 1<\alpha<1.5$

(c) $f'(x)=e^x+3$

By NRM: $x'=x-\frac{f(x)}{f'(x)}=x-\frac{e^x+3x-7}{e^x+3}$

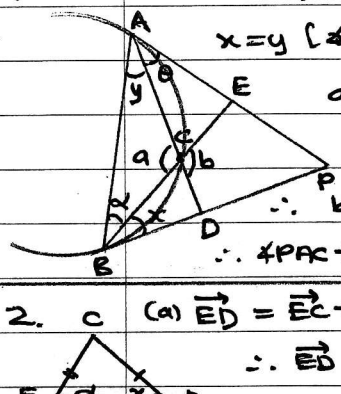
Let $x_0=1.2$: $x_1=1.2-\frac{e^{1.2}+3.6-7}{e^{1.2}+3}$

$\therefore\ x_1=1.21264\ \simeq 1.213$

$x_2=1.21260\ \simeq 1.213$

$\therefore\ \alpha=1.213$ (3 dp) ✱

1. $\theta=\alpha$ [∠ of alternate segment]




$x=y$ [∠ of alternate segment]

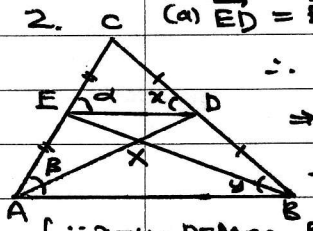
$a=b$ [opposite ∠]

$a+\alpha+y=180°$ [∠ in Δ]

$\therefore\ b+\theta+x=180°$

$\therefore\ \angle PAC+\angle PBC+\angle DCE=180°$ ✱

2. (a) $\overrightarrow{ED}=\overrightarrow{EC}+\overrightarrow{CD}=\frac{1}{2}[\overrightarrow{AC}+\overrightarrow{CB}]$




$\therefore\ \overrightarrow{ED}=\frac{1}{2}\overrightarrow{AB}\Rightarrow \overrightarrow{ED}\parallel\overrightarrow{AB}$

$\Rightarrow \alpha=B,\ x=y$ (corresponding angles)

$\frac{\text{Area }\Delta DEC}{\text{Area }\Delta ABD}=\frac{\frac{1}{2}(DE)(DC)\sin x}{\frac{1}{2}(BD)(AB)\sin y}$

$[\because x=y,\ D=M_{BC},\ ED=\frac{1}{2}AB]\quad =\frac{1}{2}$

$\therefore$ Area $\Delta DEC=\frac{1}{2}$[Area $\Delta ABD$] ✱

(b) By the same method:

Area $\Delta EDC=\frac{1}{2}$[Area $\Delta BAE$]

$\therefore$ Area $\Delta ABD$ = Area $\Delta ABD$

$\therefore\ A_{ABX}+A_{AXE}=A_{ABX}+A_{BXD}$

$\therefore$ Area $\Delta AXE$ = Area $\Delta BXD$ ✱

3. (a) $\overrightarrow{AB}.\overrightarrow{AC}=\binom{2}{-2}.\binom{3}{3}=6-6=0$

$\therefore\ AB\perp AC$ ✱

(b) $\overrightarrow{AC}=3\binom{1}{1}$, $\overrightarrow{BD}=\binom{4}{4}=4\binom{1}{1}$

$\therefore\ \overrightarrow{BD}=4(\frac{1}{3}\overrightarrow{AC})=\frac{4}{3}\overrightarrow{AC}$

$\Rightarrow\ BD\parallel AC$ ✱

(c) $\overrightarrow{BC}.\overrightarrow{BD}=\binom{1}{5}.\binom{4}{4}=\sqrt{26}.4\sqrt{2}\cos\theta$

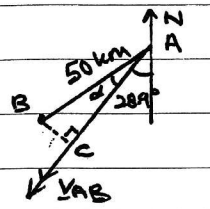
$\therefore\ \cos\theta=\frac{4+20}{4\sqrt{52}}\Rightarrow\theta=33.7°$

$\therefore\ \angle CBD=33.7°$ ✱

4. $\underline{V}_{AB}=15\binom{\sin45°}{-\cos45°}-20\binom{\sin75°}{\cos75°}=\binom{-8.712}{-15.783}$

$=\sqrt{(-8.712)^2+(15.783)^2}$ kmh$^{-1}$, to S $\tan^{-1}(\frac{8.712}{15.783})$W

$=18.03$ kmh$^{-1}$, to S 28.9° W. ✱




$SD=50\sin\alpha$

$=50\sin[40°-28.9°]$

$=9.626$ km ✱

5. $\frac{1}{\sin2x}+\frac{1}{\tan2x}=\frac{1+\cos2x}{\sin2x}=\frac{2\cos^2x}{2\cos x\sin x}=\cot x$ ✱

(a) $x=15°$: $\cot15°=\frac{1+\sqrt{3}/2}{1/2}=2+\sqrt{3}$ ✱

(b) $\frac{1}{\sin3\theta}+\frac{1}{\tan3\theta}=2\Rightarrow\cot\frac{3\theta}{2}=2$

$\therefore\ \frac{3\theta}{2}=26.57°,\ 206.57°$

$\therefore\ \theta=17.7°,\ 137.7°$ ✱

6. $\frac{dp}{dt}\propto p(1-p)\Rightarrow\frac{dp}{dt}=kp(1-p)$ ✱

$\frac{d^2p}{dt^2}=0$: $k(1-2p)=0\Rightarrow p=\frac{1}{2}$

$\frac{d^3p}{dt^3}=-2k\ (<0)$ $\therefore$ Growth max. when $p=\frac{1}{2}P$ ✱

$\int\frac{dp}{p(1-p)}=\int k\,dt\Rightarrow\int[\frac{1}{1-p}+\frac{1}{p}]dp=kt+c$

$\therefore\ \ln(\frac{p}{1-p})=kt+c$ —①

$t=0,\ p=\frac{1}{4}$: $\ln\frac{1}{3}=c$ —②

①−②: $\ln[\frac{3p}{1-p}]=kt$

$t=20,\ p=\frac{1}{2}$: $\ln3=20k$

$\therefore\ p=\frac{7}{8}$: $t=\ln21/\frac{1}{20}\ln3=55.42$ years ✱

max growth $=\frac{1}{20}\ln3.(\frac{1}{2})(\frac{1}{2})=0.014$ ✱

7. $P(D\geq17.15)=P(Z\geq\frac{17.15-17.1}{0.04})=0.1057$

$\therefore$ No. of qualify $=0.1057\times15=1.5855$

$\approx2$ ✱

8. (a) P(all meet) = P(all Rex)

$=(\frac{1}{2})(\frac{1}{2})(\frac{1}{2})^2=\frac{1}{16}$ ✱

(b) P(All diff) = $P(A_RB_VC_C)+P(A_CB_VC_R)$

$+P(A_CB_RC_V)=\frac{1}{2}(\frac{1}{2})(\frac{1}{2})+(\frac{1}{2})(\frac{1}{2})^2\times2$

$=\frac{1}{4}$ ✱

(c) P(At least 2 meet)

$=1-(\frac{1}{16}+\frac{1}{4})=\frac{11}{16}$ ✱

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9. $X$ = no. of people enter in a minutes

$X \sim Po(2)$

(a) $P(X=5) = e^{-2}\left[\frac{2^5}{5!}\right] = 0.0361$ *

(b) $P(X_1+X_2<4)$ $[X_1+X_2 \sim Po(4)]$

$= e^{-4}\left[1+4+\frac{4^2}{2!}+\frac{4^3}{3!}\right] = 0.4335$ *

10. (a) $\frac{1}{2}(2)[2k+0]=1 \Rightarrow k=\frac{1}{2}$ *

(b) $P(X\le x) = \int_0^x \frac{1}{2}(2-x)\,dx$

$= -\frac{1}{4}\left[(2-x)^2\right]_0^x = \frac{1}{4}\left[4-(2-x)^2\right]$

$\therefore F(x) = \begin{cases} 0 & x<0 \\ \frac{1}{4}(4-(2-x)^2) & 0\le x\le 2 \\ 1 & 2<x \end{cases}$ *

$Y = 1-\frac{X}{2}$: $P(Y<y) = P(1-\frac{X}{2}<y)$

$= P(X>2(1-y)) = 1-P(X\le 2(1-y))$

$= 1-\frac{1}{4}\left[4-(2-2(1-y))^2\right]$ for $0\le 2(1-y)\le 2$

$= y^2$ for $0\le y\le 1$ *

11. (a) $\Sigma P(X=x)=1$: $p+q+0.5=1$

$\therefore p+q=0.5$ —①

$E(X)=4$: $2p+0.6+1.2+5q=4$

$\therefore 2(0.5)+1.8+3q=4$

$\therefore q=0.4$, $p=0.1$ *

(b) $E(X^2) = 4(0.1)+1.8+4.8+25(0.4)$

$= 17.$

$Var(X) = 17-(4)^2 = 1$ *

(c) $E(|X-4|) = 2(0.1)+1(0.2)+0$

$+ 1(0.4) = 0.8$ *

12. (a) $n=25$, $\Sigma x = 260$, $\Sigma x^2 = 3568$

$\bar{x} = \frac{260}{25} = 10.4$ *

$s = \left[\frac{3568}{25} - 10.4^2\right]^{1/2} = 5.879$ *

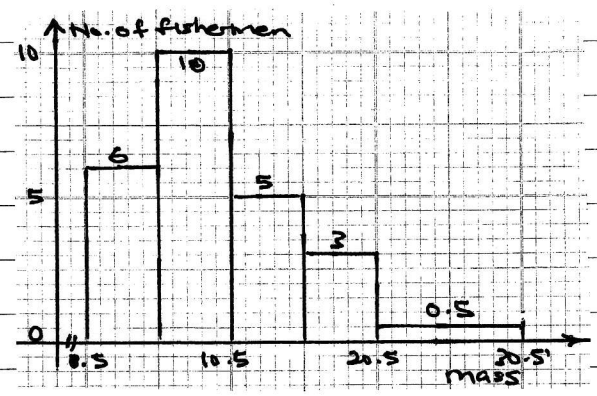
(b) Range is from $\bar{x}$ $= (\bar{x}-s, \bar{x}+s)$

$= (4.521, 16.279)$

$\therefore$ Percentage $= \frac{18}{25}\times 100\% = 72\%$ *

(c)

| masses | f |
|---|---|
| $0.5\le x<5.5$ | 6 |
| $5.5\le x<10.5$ | 10 |
| $10.5\le x<15.5$ | 5 |
| $15.5\le x<20.5$ | 3 |
| $20.5\le x<30.5$ | 1 |

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