LEE HOCK LEONG
M.Sc. (Pure Maths) – CGPA 4
B.Sc with Ed. (1st Class Hons.)

PERLIS (2006)
MATHEMATICS T (PAPER 1)

1. If $z = \frac{\sqrt{3} + ai}{1 + \sqrt{3}\,i}$ is a real number, determine the value of a and z. [4]

2. Given that α and β are roots of the quadratic equation $2x^2 + 3x + 4 = 0$.
   Find the values of
   (a) $\alpha - \beta$ [3]
   (b) $(\sqrt{\alpha} - \sqrt{\beta})^2$ [3]
   (c) $\frac{\alpha^2}{2\beta - 3} + \frac{\beta^2}{2\alpha - 3}$ [5]

3. Find the set of values of x such that $\frac{3(x-2)}{4} + \frac{1}{x+2} > \frac{1}{x-1}$. [7]

4. Express $\sqrt{\frac{1+2x}{1+x}}$ as a series of ascending powers of x up to the term in x^3. [5]
   By taking $x = \frac{1}{3}$, find $\sqrt{5}$ correct to four decimal places. [3]

5. (a) Determine the smallest integer n such that $1 + \frac{4}{3} + \left(\frac{4}{3}\right)^2 + \ldots + \left(\frac{4}{3}\right)^n > 30$. [5]
   (b) Find the sum to infinity of
   $$2^2(x-1)^2 + 2^3(x-1)^3 + 2^4(x-1)^4 + \ldots + 2^r(x-1)^r + \ldots$$
   and determine the value of x such that this sum exists. [5]

6. Given that $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{pmatrix}$. Show that $\mathbf{A}^{-1}$ exists and find $\mathbf{A}^{-1}$. [5]
   Hence, solve the following simultaneous equation using matrix equation.
   $x + 2y + 3z = 12$, $2x + 3y + z = 7$ and $3x + y + 2z = 5$. [4]

7. Find $\int \frac{2}{2+x^2}\,dx$ by using the substitution $x = \sqrt{2}\tan\theta$. [5]

8. A curve is defined by the parametric equations $x = 1 - 2t$ and $y = \frac{2}{t} - 2$.
   Find the equation of the normal to the curve at the point $A(0, 2)$. [6]
   The normal to the curve at the point $A(0, 2)$ cuts the curve again at the point B.
   Find the coordinates of B. [3]

9. Sketch, on the same coordinates axes, the graphs of $y = e^x$ and $y = \frac{2}{1+x}$.
   Show that the equation $(1 + x)e^x - 2 = 0$ has a root in the interval $[0, 1]$. [5]
   By using Newton-Raphson method with the initial estimate $x_0 = 0.4$, find the estimation for the root, correct to three decimal places. [5]

10. If $y = \cos[\ln(1 + x)]$, show that $(1 + x)^2 \frac{d^2y}{dx^2} + (1 + x)\frac{dy}{dx} + y = 0$. [5]

11. Express $\frac{2x-1}{(2+x)(1+x^2)}$ as partial fractions. [4]
    Hence, evaluate $\int_{-1}^{0} \frac{2x-1}{(2+x)(1+x^2)}\,dx$. [5]

12. Given that $f(x) = \begin{cases} x^2 & , x > 0 \\ 2x & , x \le 0 \end{cases}$ and $g(x) = \begin{cases} \sqrt{x} & , x \ge 0 \\ -\frac{x}{2} & , x < 0 \end{cases}$.
    (a) State the ranges of f and g. [2]
    (b) Determine whether f and g is a one to one function. [2]
    (b) Define (i) f^{-1} [3]
    (ii) gf [3]
    (iii) fg [3]

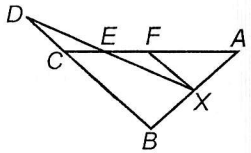
PERLIS (2006)
MATHEMATICS T (PAPER 2)

1. Solve the differential equation
   $(1 - \cos 2x)\frac{dy}{dx} - 2(1 + y)\sin 2x = 0$, given that $y = 0$ when $x = \frac{\pi}{2}$. [5]

2. Prove the following identities,
   (a) $\frac{\cot\theta}{\cot\theta - \cot 3\theta} + \frac{\tan\theta}{\tan\theta - \tan 3\theta} = 1$ [4]
   (b) $\frac{\cos(\theta - 3\phi) - \cos(3\theta - \phi)}{\sin(3\theta + \phi) + \sin(\theta - 3\phi)} = \tan(\theta + 2\phi)$ [5]

3. Solve the equation $3\cos\theta - 8\sin\theta = -2$, for $0^\circ \le \theta \le 360^\circ$. [6]

4. Given that $AB = BC$, $AX = CD$ and FX is parallel to DB.
Prove that $FX = CD$ and $XE = DE$. [8]



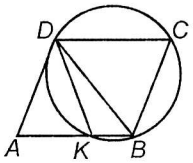
5. An airplane A which can fly at a maximum speed of $800\ \text{km h}^{-1}$ is going to intercept airplane B. Airplane B is fling to the east with a speed of $500\ \text{km h}^{-1}$. Initially, B is at a position 100 km and on a bearing 240° from A.

(a) Find the direction of A for which it can intercept B as fast as possible. [6]

(b) Find the minimum speed required for A to intercept B. [2]

(c) If A flies at a speed of $400\ \text{km h}^{-1}$, find the two possible directions of A for which A will be able to intercept B. [4]

6. $ABCD$ is a parallelogram, and AD is a tangent to the circle at D. If AKB is a straight line, prove that $AD = DK$ and $AB = BD$. [8]



7. Find the value of λ for which the vectors $\underline{u} = 3\underline{i} + 2\underline{j}$ and $\underline{v} = 2\underline{i} + \lambda\underline{j}$ are perpendicular. [3]

8. The number of personal computers sold in a week by a company for a period of 25 weeks is given as follows:

10	5	7	4	4	18	9	14	17	11	8	4
5	7	10	12	14	10	9	29	7	6	5	20	15

(a) Find the mean and the standard deviation of the number of personal computers sold in a week in a given period. [3]

(b) Find the percentage of the weeks that the weekly sales of the personal computers are within one standard deviation from the mean. [3]

(c) Find the median, lower quartile and upper quartile for this distribution. Hence, draw a box plot to represent the data, and state the shape of the distribution. [6]

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9. A box contains 48 books as shown in the table.

	Red	Blue	Yellow	White
Mathematics	6	4	10	0
Accountancy	4	12	4	8

Three books are selected randomly from the box.

Events X, Y and Z are defined as follow,

X: no white books are selected

Y: the books selected included an Accountancy book of red colour

Z: at least a Mathematics book is selected

Find $P(X)$, $P(Y)$, $P(X|Y)$, $P(X \cup Y)$, $P(Z)$ and $P(Z|X)$. [10]

10. A discrete random variable X has the following probability distribution.

x	1	2	3	4
$P(X = x)$	c	c^2	$c^2 + c$	$2c^2 + c$

where c is a constant.

(a) Find the value of c. [3]

(b) Sketch the probability distribution. [2]

11. The cumulative distribution function $F(x)$ of a continuous random variable X is defined as,

$$F(x) = \begin{cases} 0 & , x < 2 \\ \frac{x^2}{6} - \frac{2x}{3} + a & , 2 \le x < 3 \\ b + \frac{x}{3} & , 3 \le x < 5 \\ 2x - \frac{x^2}{6} + c & , 5 \le x < 6 \\ 1 & , 6 \le x \end{cases}$$

where a, b and c are constants.

(a) Find the values of a, b and c. [4]

(b) Find the probability density function, $f(x)$. [3]

(c) Sketch the graph of f. [2]

12. The chocolate cakes baked by SJ cake house are supposed to weigh 1 kg each. The weights of the chocolate cake are normally distributed with mean 1.03 kg and standard deviation 0.017 kg.

(a) Find the percentage of cakes with weight less than 1 kg. [2]

(b) Find the probability that if five cakes are randomly chosen, at least two cakes weight more than 1 kg. [4]

(c) Calculate the probability that four randomly chosen cakes has a total weight of more than 4.2 kg. [4]

(d) It is desired to adjust the mean weight of the chocolate cake such that at least 92% of the chocolate cakes will have weight not less than 1 kg while keeping the standard deviation at 0.017 kg.
Find the least value to which the mean weight should be set. [3]

PERLIS (2006)

MT1

No.: ..... Date: .....

1. $z = \dfrac{\sqrt{3}+ai}{1+\sqrt{3}i}$ [Given that $z \in \mathbb{R}$]

$\therefore\ z + \sqrt{3}zi = \sqrt{3} + ai$

Re: $z = \sqrt{3}$ ✱ Im: $a = \sqrt{3}(\sqrt{3}) = 3$ ✱

2. $2x^2+3x+4=0$ [Roots $= \alpha, \beta$]

$\therefore\ \alpha+\beta = -\frac{3}{2}$, $\alpha\beta = 2$

(a) $\alpha-\beta = \sqrt{(\alpha-\beta)^2} = [(\alpha+\beta)^2 - 4\alpha\beta]^{1/2}$

$= [\frac{9}{4} - 8]^{1/2} = [-\frac{23}{4}]^{1/2} = \frac{\sqrt{23}}{2}i$ ✱

(b) $(\sqrt{\alpha}-\sqrt{\beta})^2 = \alpha+\beta-2\sqrt{\alpha\beta} = -[\frac{3}{2}+2\sqrt{2}]$ ✱

(c) $\dfrac{\alpha^2}{2\beta-3} + \dfrac{\beta^2}{2\alpha-3} = \dfrac{-(3\alpha+4)}{2(2\beta-3)} - \dfrac{(3\beta+4)}{2(2\alpha-3)}$

$= -\left[\dfrac{6(\alpha^2+\beta^2) - (\alpha+\beta) - 24}{2(4\alpha\beta - 6(\alpha+\beta)+9)}\right]$

$= -\left[6(\frac{9}{4}) - 6(2) - (-\frac{3}{2}) - 24\right] / 2[8+9+9] = \frac{21}{52}$ ✱

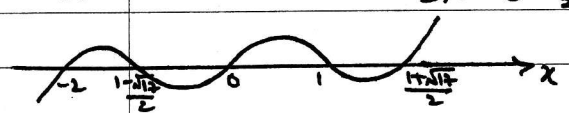
3. $\dfrac{3(x-2)}{4} + \dfrac{1}{x+2} > \dfrac{1}{x-1}$ — $\times 4(x+2)^2(x-1)^2$

$\therefore\ 3(x-2)(x+2)^2(x-1)^2 + 4(x+2)(x-1)^2 > 4(x+2)^2(x-1)$

$\therefore\ (x+2)(x-1)[3x^3-3x^2-12x+12+4x-4-4x-8] > 0$

$\therefore\ (x+2)(x-1)(3)(x)(x^2-x-4) > 0$

$\therefore\ (x+2)(x-1)(x)(x-\frac{1}{2}-\frac{\sqrt{17}}{2})(x-\frac{1}{2}+\frac{\sqrt{17}}{2}) > 0$



$\therefore\ \{x : -2 < x < \frac{1-\sqrt{17}}{2}$ or $0 < x < 1$ or $\frac{1+\sqrt{17}}{2} < x\}$ ✱

4. $\sqrt{\dfrac{4-2x}{1+x}} = f(x) = [1+\frac{1}{2}(2x)+\frac{1}{2}(\frac{1}{2})(-\frac{1}{2})(2x)^2+$

$\frac{1}{6}(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})(2x)^3+\dots][1+(-\frac{1}{2})(x)+$

$\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})(x)^2+\frac{1}{6}(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})(x)^3+\dots]$

$= [1+x-\frac{1}{2}x^2+\frac{1}{2}x^3+\dots][1-\frac{1}{2}x+\frac{3}{8}x^2-\frac{5}{16}x^3+\dots]$

$= 1+\frac{1}{2}x-\frac{5}{8}x^2+\frac{13}{16}x^3+\dots$ ✱

$f(\frac{1}{3}) = \sqrt{\frac{5}{3}/\frac{4}{3}} = \frac{\sqrt{5}}{2}$

$\therefore\ \sqrt{5} = 2[1+\frac{1}{2}(\frac{1}{3})-\frac{5}{8}(\frac{1}{3})^2+\frac{13}{16}(\frac{1}{3})^3+\dots]$

$= 2.2546$ (4dp) ✱

5 (a) $1+\frac{4}{3}+(\frac{4}{3})^2+\dots+(\frac{4}{3})^n > 30$

$\therefore\ \frac{4}{3}[(\frac{4}{3})^n - 1 / \frac{4}{3}-1] > 29$

$\therefore\ (\frac{4}{3})^n > \frac{33}{4}$

$\therefore\ n > \lg(\frac{33}{4}) / \lg(\frac{4}{3}) = 7.34$

$\therefore\ n_{min} = 8$ ✱

(b) $2^2(x-1)^2 + 2^3(x-1)^3 + \dots = S_\infty$

$= 2^2(x-1)^2 / 1-2(x-1) = \dfrac{4(x-1)^2}{3-2x}$ ✱

For S_∞ exists: $|2(x-1)| < 1$

$\therefore\ -\frac{1}{2} < x-1 < \frac{1}{2} \Rightarrow \{x : \frac{1}{2} < x < \frac{3}{2}\}$ ✱

6. $|A| = 1(6-1) - 2(4-3) + 3(2-9) = -18\ (\neq 0)$

$\therefore\ A^{-1}$ exists ✱

$C_A = \begin{pmatrix} 6-1 & -(4-3) & 2-9 \\ -(4-3) & 2-9 & -(1-6) \\ 2-9 & -(1-6) & 3-4 \end{pmatrix} = \begin{pmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{pmatrix}$

$\therefore\ A^{-1} = \frac{1}{-18}(C_A)^T = -\frac{1}{18}\begin{pmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{pmatrix}$ ✱

$\left.\begin{array}{l} x+2y+3z=12 \\ 2x+3y+z=7 \\ 3x+y+2z=5 \end{array}\right\}$ $A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 \\ 7 \\ 5 \end{pmatrix}$

$\therefore\ \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{1}{18}\begin{pmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{pmatrix}\begin{pmatrix} 12 \\ 7 \\ 5 \end{pmatrix}$

$\therefore\ \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \Rightarrow x=-1, y=2, z=3$ ✱

7. $x = \sqrt{2}\tan\theta \Rightarrow \frac{dx}{d\theta} = \sqrt{2}\sec^2\theta$

$\int \frac{2}{2+x^2}dx = \int \frac{2}{2(1+\tan^2\theta)} \cdot \sqrt{2}\sec^2\theta\, d\theta$

$= \int \sqrt{2}\, d\theta = \sqrt{2}\theta + c = \sqrt{2}\tan^{-1}(\frac{x}{\sqrt{2}}) + c$ ✱

8. $x = 1-2t \Rightarrow \frac{dx}{dt} = -2$

$y = \frac{2}{t} - 2 \Rightarrow \frac{dy}{dt} = -\frac{2}{t^2}$

$\frac{dy}{dx} = \frac{-2}{t^2} / -2 = \frac{1}{t^2}$

A(0,2): $1-2t=0 \Rightarrow t = \frac{1}{2}$

$\therefore\ \frac{dy}{dx} = 1/(\frac{1}{2})^2 = 4$

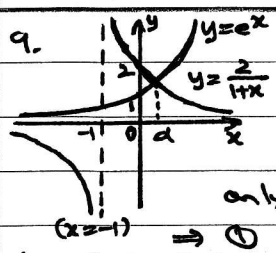
$\therefore$ Normal (A): $y = -\frac{1}{4}x + 2$ ✱

For B: $\frac{2}{t} - 2 = -\frac{1}{4}(1-2t) + 2$

$\therefore\ 2t^2+15t-8=0 \Rightarrow (2t-1)(t+8)=0$

$\therefore\ t_B = -8 \Rightarrow B(17, -\frac{9}{4})$ ✱

9.



$(1+x)e^x - 2 = 0$ — ①

$\therefore\ e^x = \frac{2}{1+x}$

$y=e^x$ and $y=\frac{2}{1+x}$ has only 1 intersection point.

$\Rightarrow$ ① has only 1 real root.

Let $f(x) = (1+x)e^x - 2$: $f'(x) = (2+x)e^x$

$f(0)\cdot f(1) = (-1)(3.44) > 0$

$\therefore$ ① has a root in the interval $[0,1]$ ✱

By NRM: $x' = x - \frac{f(x)}{f'(x)} = x - \frac{1+x-2e^{-x}}{2+x}$

$x_0 = 0.4$: $x_1 = 0.4 - \frac{1+0.4-2e^{-0.4}}{2+0.4}$

$= 0.37527 \approx 0.375$

$x_2 = 0.37482 \approx 0.375$

$\therefore\ \alpha = 0.375$ (3dp) ✱

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PERLIS (2006)
MT1 / MT2

Date:

10. $y = \cos[\ln(1+x)] \Rightarrow y' = \dfrac{-\sin[\ln(1+x)]}{1+x}$

$\therefore (1+x)^2(y')^2 = \sin^2[\ln(1+x)] = 1-y^2$

$\therefore (1+x)^2(2)y'y'' + 2(1+x)(y')^2 = -2yy'$

$\therefore (1+x^2)y'' + (1+x)y' + y = 0$ ✱

11. $f(x) = \dfrac{2x-1}{(2+x)(1+x^2)} \equiv \dfrac{A}{2+x} + \dfrac{Bx+C}{1+x^2}$

$\equiv \dfrac{A(1+x^2)+(Bx+C)(2+x)}{(2+x)(1+x^2)}$

$\therefore 2x-1 \equiv A(1+x^2)+(Bx+C)(2+x)$

$\underline{x=-2}$: $-5 = 5A \Rightarrow A=-1$

$\underline{[x^2]}$: $0 = A+B \Rightarrow B=1$

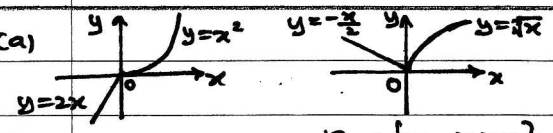
$\underline{[x^0]}$: $-1 = A+2C \Rightarrow C=0$

$\therefore f(x) = \dfrac{x}{1+x^2} - \dfrac{1}{2+x}$ ✱

$\int_{-1}^{0} f(x)\,dx = \left[\frac{1}{2}\ln(1+x^2) - \ln(2+x)\right]_{-1}^{0}$

$= [0-\ln 2] - [\frac{1}{2}\ln 2 - 0] = -\frac{3}{2}\ln 2$ ✱

12.(a)



$\underline{R_f}$: $x \in \mathbb{R}$ ✱ $\qquad R_g$: $\{x : x>0\}$ ✱

(b) f is 1-1, g is many-1 ✱

(c) (i) $f^{-1}: x \to \begin{cases} x/2, & x \le 0 \\ \sqrt{x}, & x>0 \end{cases}$ ✱

(ii) $gf(x) = \begin{cases} g(x^2), & x>0 \\ g(2x), & x \le 0 \end{cases}$

$= \begin{cases} x, & x>0 \\ -x, & x \le 0 \end{cases}$ ✱

(iii) $fg(x) = \begin{cases} f(\sqrt{x}), & x>0 \\ f(-x/2), & x \le 0 \end{cases}$ ✱

$= \begin{cases} x, & x>0 \\ x^2/4, & x \le 0 \end{cases}$ ✱

1. $(1-\cos 2x)\dfrac{dy}{dx} - 2(1+y)\sin 2x = 0$

$\therefore \int \dfrac{1}{1+y}\,dy = \int \dfrac{2\sin 2x}{1-\cos 2x}\,dx$

$\therefore \ln(1+y) = \ln(1-\cos 2x) + c$

$\underline{x=\frac{\pi}{2}, y=0}$: $0 = 0 + c$

$\therefore \ln(1+y) = \ln(1-\cos 2x)$

$1+y = 1-\cos 2x \Rightarrow y = -\cos 2x$ ✱

2.(a) $\dfrac{\cot\theta}{\cot\theta - \cot 3\theta} + \dfrac{\tan\theta}{\tan\theta - \tan 3\theta}$

$= \dfrac{1}{\tan\theta} \Big/ \left[\dfrac{1}{\tan\theta} - \dfrac{1}{\tan 3\theta}\right] + \dfrac{\tan\theta}{\tan\theta - \tan 3\theta}$

$= \dfrac{-\tan 3\theta}{\tan\theta - \tan 3\theta} + \dfrac{\tan\theta}{\tan\theta - \tan 3\theta} = 1$ ✱

(b) $\dfrac{\cos(\theta-3\phi) - \cos(3\theta+\phi)}{\sin(3\theta+\phi) + \sin(\theta-3\phi)}$

$= \dfrac{-2\sin(2\theta-\phi)\sin(-\theta-2\phi)}{2\sin(2\theta-\phi)\cos(\theta+2\phi)} = \dfrac{-[-\sin(\theta+2\phi)]}{\cos(\theta+2\phi)}$

$= \tan(\theta+2\phi)$ ✱

3. Let $t = \tan\frac{\theta}{2}$:

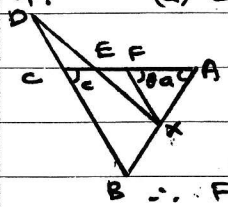
$3\cos\theta - 8\sin\theta = -2 \Rightarrow 3\left(\frac{1-t^2}{1+t^2}\right) - 8\left(\frac{2t}{1+t^2}\right) = -2$

$\therefore t^2 - 16t - 5 = 0 \Rightarrow t = \dfrac{16 \pm \sqrt{256+20}}{2}$

$\therefore \frac{\theta}{2} = 86.49°$ or $162.95°$

$\therefore \theta = 173.0°$ or $325.9°$ ✱

4. (a) $c = a$ [AB=BC, isosceles Δ]



$c = \theta$ [corresponding ∡ FX // DB]

$\therefore a = \theta \Rightarrow \Delta AFX$ is isosceles with FX = AX

AX = CD [given]

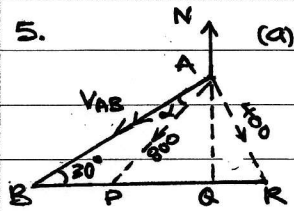
$\therefore FX = CD$ ✱

(b) $\left.\begin{array}{l} \measuredangle CDE = \measuredangle FXE \\ \measuredangle DCE = \measuredangle XFE \end{array}\right\}$ Alternate angles FX // DB

$\therefore \Delta CDE \equiv \Delta FXE$ (by ASA)

$\Rightarrow XE = DE$ ✱

5.



(a) $\underline{\Delta ABP}$: $\dfrac{\sin\alpha}{500} = \dfrac{\sin 30°}{800}$

$\therefore \sin\alpha = \frac{5}{16} \Rightarrow \alpha = 18.2°$

$\therefore$ Direction of A = S41.8°W ✱

time taken $= \dfrac{100}{500\cos 30° + 800\cos 18.2°}$ h ≈ 5 min ✱

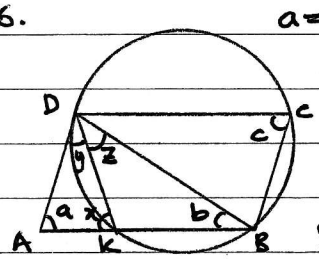
(b) $\underline{\Delta ABQ}$: $|V_A|_{min} = 500\tan 30° = 288.7\,kmh^{-1}$ ✱

(c) $\underline{\Delta ABR}$: $\dfrac{\sin\measuredangle BAR}{500} = \dfrac{\sin 30°}{400}$

$\therefore \sin\measuredangle BAR = \frac{5}{8} \Rightarrow \measuredangle BAR = 38.7°$ or $141.3°$

$\therefore$ Direction of A = S21.3°W or S21.3°E ✱

6.



$a = c$ [opposite angles of parallelogram]

$c = x$ [Ext. ∡ of cyclic KBCD]

$\therefore a = x$

$\therefore AD = DK$ [ΔDAK is isosceles] ✱

$y = b$ [∡ of alternate segment]

$b + z = x$ [ext. ∡ of ΔBKD]

$\therefore y + z = a \Rightarrow \Delta BAD$ is isosceles with AB = BD ✱

7. $\underline{u} \perp \underline{v}$: $\begin{pmatrix}3\\2\end{pmatrix}\cdot\begin{pmatrix}2\\\lambda\end{pmatrix}=0$

$6+2\lambda=0 \Rightarrow \lambda=-3$ *

8. 44455 56777 899 10 10
10 11 12 14 14 15 17 18 20 29

(a) $n=25,\ \Sigma x=260 \quad \Sigma x^2=3568$

$\bar{x}=\frac{260}{25}=10.4$ *

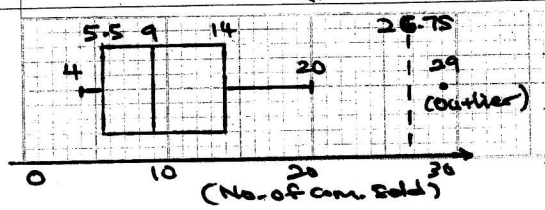
$s=\left[\frac{3568}{25}-10.4^2\right]^{1/2}=5.879$ *

(b) Range $=(\bar{x}-s,\ \bar{x}+s)=(4.521, 16.279)$

$\therefore$ Percentage $=\frac{18}{25}\times 100\% = 72\%$ *

(c) $m=x_{13}=9$ * $\quad Q_1=\frac{1}{2}(5+6)=5.5$ *

$Q_3=\frac{1}{2}(14+14)=14$ *



Distribution: right-skewed *

9. $P(X)=\ {}^{40}C_3 / {}^{48}C_3=\frac{1235}{2162}$ *

$P(Y)=\frac{{}^4C_1\cdot{}^{44}C_2}{{}^{48}C_3}=\frac{473}{2162}$ *

$P(X|Y)=\frac{P(X\cap Y)}{P(Y)}=\frac{{}^4C_1\cdot{}^{36}C_2}{{}^{48}C_3}\Big/\frac{473}{2162}=\frac{315}{473}$ *

$P(X\cup Y)=\frac{1235+473-315}{2162}=\frac{1393}{2162}$ *

$P(Z)=1-\frac{{}^{28}C_3}{{}^{48}C_3}=\frac{3505}{4324}$ *

$P(Z|X)=\frac{P(Z\cap X)}{P(X)}=\frac{{}^{20}C_1\cdot{}^{20}C_2+{}^{20}C_2\cdot{}^{20}C_1+{}^{20}C_3}{{}^{40}C_3}$

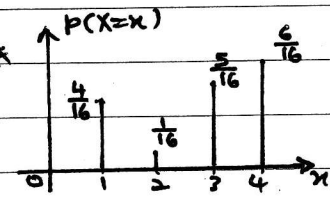
$=\frac{23}{26}$ *

10. (a) $\Sigma P(X=x)=1$: $4c^2+3c=1$

$\therefore 4c^2+3c-1=0 \Rightarrow (4c-1)(c+1)=0$

$[\because c>0]$: $c=\frac{1}{4}$ *

(b)



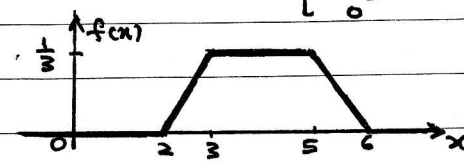
11(a) $F(2)=\lim_{x\to 2^-}F(x)$: $\frac{2}{3}-\frac{4}{3}+a=0 \Rightarrow a=\frac{2}{3}$ *

$F(3)=\lim_{x\to 3^-}F(x)$: $\frac{3}{2}-2+\frac{2}{3}=b+1 \Rightarrow b=-\frac{5}{6}$ *

$F(6)=\lim_{x\to 6^-}F(x)$: $1=12-6+c \Rightarrow c=-5$ *

(b) $f(x)=\frac{d}{dx}[F(x)]=\begin{cases}\frac{1}{3}(x-2) & 2\le x<3\\ \frac{1}{3} & 3\le x<5\\ 2-\frac{1}{3}x & 5\le x<6\\ 0 & \text{otherwise}\end{cases}$ *

(c)



12. W = weight of coke, $W\sim N(1.03, 0.017^2)$

(a) $P(W<1)=P\left(Z<\frac{1-1.03}{0.017}\right)=0.03881$

$\therefore$ Percentage $(<1)=3.881\%$ *

(b) Prob $=1-\left[(0.03881)^5+{}^5C_1(0.03881)^4(0.96119)\right]$

$=0.99999$ *

(c) $T=W_1+W_2+W_3+W_4$:

$T\sim N(4(1.03),\ 4(0.017)^2)$

$P(T>4.2)=P\left(Z>\frac{4.2-4.12}{2(0.017)}\right)=0.00931$ *

(d) $P(X\ge 1)\ge 0.92$:

$P\left(Z\ge\frac{1-\mu}{0.017}\right)\ge 0.92 \Rightarrow \frac{1-\mu}{0.017}\le -Z_{0.08}$

$\therefore \frac{1-\mu}{0.017}\le -1.405 \Rightarrow \mu\ge 1.0239$

$\therefore \mu_{min}=1.0239$ *

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